

Lesson 4

GCSE : Algebra of Inequalities

4.1 Multiplication By A Negative Number

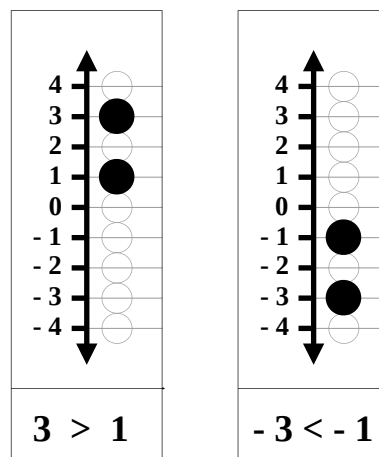
The inequalities looked at so far have behaved in the same manner as equations.

For Example;

Finding the x from the inequality $8x + 9 < 33$ is done in a very similar manner to finding the x from the equation $8x + 9 = 33$.

$$\begin{array}{rcl} 8x + 9 < 33 & | & \\ 8x < 24 & | & -9 \\ x < 3 & | & \div 8 \end{array} \qquad \begin{array}{rcl} 8x + 9 = 33 & | & \\ 8x = 24 & | & -9 \\ x = 3 & | & \div 8 \end{array}$$

There is a complication that arises when solving some inequalities.
To understand the complication think about this diagram;



IMPORTANT RESULT

If we multiply (or divide) both sides of an inequality by a negative number then we must reverse the direction of the inequality

IMPLICATION

You cannot multiply (or divide) both sides of an inequality by x when the sign of x is not known because you would not know if to reverse the inequality or not.

4.2 Three Questions

Solve the following inequalities;

(i) $21 - x < 17$

(ii) $43 + 4x \geq 23$

(iii) $31 - 3x > 88$

4.3 Answers to The Three Questions

(i) $x > 4$

(ii) $x \geq -5$

(iii) $x < -19$

4.4 Exercise : Inequalities That Sometimes Need Reversing

Do NOT use a calculator

Question 1

Solve the following inequalities;

(i) $4x < 12$

(ii) $-5x \geq 35$

(iii) $-3x > -15$

(iv) $7x > -28$

(v) $-6x \leq 3$

(vi) $10x > -4$

(vii) $-x \geq 1$

(viii) $4 - x \geq 11$

(ix) $4 > x$

(x) $-4 > x$

(xi) $7x \geq -3.5$

(xii) $-2x + 7 > 5$

Question 2

Solve the following inequalities;

(i) $5x + 7 < 22$

(ii) $8 + 6x > -10$

(iii) $-7 + 2x \geq 13$

(iv) $14 - 3x \leq 5$

(v) $3(x - 5) > 9$

(vi) $0.4x + 2.4 < 4.8$

Question 3

In a field are a lot of sheep.

I'm not sure exactly how many.

Let's say that there are x sheep in the field.

224 sheep are taken out of the field.

The number of sheep now in the field is less than 100.

Write down, and solve, an inequality for x .

Question 4

Let A be the number of prime numbers less than 25.

Let B be the number of prime numbers less than 101.

Which one of the following statements is true ?

(a) $A > B$

(b) $A < B$

(c) $A = B$

Question 5

If x is an *integer*, list all of the values of x for which it is true that;

$$3 < x \leq 7$$

Question 6

If x is an *integer*, list all of the values of x for which it is true that;

$$-2 \leq x < 6$$

Question 7

If x is a *positive integer*, list all of the values of x for which it is true that;

$$x \leq 4$$

Question 8

If Mr X is a teenager, which one of the following inequalities most accurately describes his possible ages.

$$12 \leq X < 19$$

$$13 < X \leq 19$$

$$12 < X \leq 19$$

$$13 \leq X < 19$$

Question 9

Two sides of a mathematician's triangle are 8 cm and 13 cm exactly.

Write down an inequality in the form

$$a \leq x \leq b$$

where x is the length of the third side, and a & b are numbers the values of which are to be stated.

Question 10

Solve the following inequalities;

(i) $\frac{4x}{3} + 5 < 13$

(ii) $23 - 3x > -1$

(iii) $7x + 9 > 3x + 37$

(iv) $38 - 3x > 65$

Question 11

Calculate;

$$(4)^2 \quad (3)^2 \quad (-1)^2 \quad \text{and} \quad (-2)^2$$

If x is an integer, list **ALL** values of x for which $x^2 \leq 9$.

Question 12

In this question x is NOT an integer.

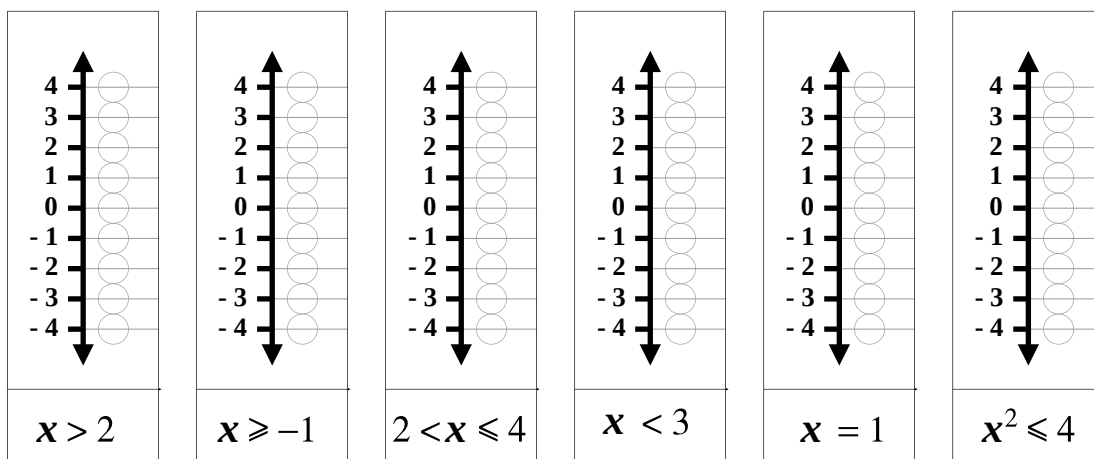
Also you are told that $x^2 < 16$.

Solve this inequality and write your answer in the form

$a < x < b$, where a and b are numbers you must find.

Question 13

On these number ladders draw the inequalities indicated;



Question 14

To the nearest cm, a plank of wood is 123 cm.

Write down an inequality of the form;

$$a \leq x < b$$

where x is the exact length of the plank, and a & b are numbers, the values of which you must state.

Out Of Interest :

a is referred to as the lower bound of the plank's length, and b the upper bound.