A-Level Pure Mathematics

Vectors II: Year 1 and Year 2

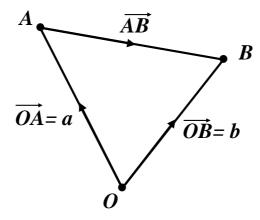
2.1 The Vector Between Two points

Statement:

$$\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$$

Proof:

The result is obvious from a study of the following diagram



A more mathematical proof is to argue as follows;

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

$$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$$

In words we say that \overrightarrow{AB} is \boldsymbol{b} relative to \boldsymbol{a} with the words "relative to" being the interpretation of the minus sign.

Interpretation:

If these were displacement vectors \overrightarrow{AB} is the position of \boldsymbol{b} relative to \boldsymbol{a} which tells you how to get to \boldsymbol{b} from \boldsymbol{a} .

2.2 Exercise

Question 1

A is (1, 4)

B is (7, 6)

Write down \overrightarrow{AB}

Question 2

C is (-3, 4)

D is (2, 3)

Write down \overrightarrow{CD}

Question 3

P is (7,3)

Q is (1, 2)

Write down \overrightarrow{PQ}

Question 4

A is (-1, 4)

B is (5, 9)

Write down \overrightarrow{AB}

Question 5

A is (-3, -5)

B is (2, 1)

Write down \overrightarrow{BA}

Question 6

A is (5, -2)

B is (3, 0)

Write down \overrightarrow{BA}

Question 7

P is (4, 1)

Q is (6,3)

Write down \overrightarrow{QP}

Question 8

A is (-1, -3)

B is (-5, -8)

Write down \overrightarrow{AB}

P is the point (6,5) and Q is the point (-3,3)

Determine the vector \overrightarrow{PQ}

Does your vector take you from P to Q or from Q to P?

(Draw a sketch of the situation to convince yourself that your answer is correct)

Question 10

M is the point (7, -4) and N is the point (11, 8)

Determine the vector \overrightarrow{MN}

Have you worked out the position of M relative to N or of N relative to M? (Draw a sketch of the situation to convince yourself that your answer is correct)

Question 11

C is the point (-7, 12) and D is the point (8, -3)Determine the position of C relative to D

At the start of a walk, I am at the position given by $r_A = 1.3 i + 0.4 j$ km I walk directly, in a straight line, to $r_B = 0.3 i - 0.7 j$ km

- (i) Determine the vector that describes my walk.
- (ii) By using the theorem of Pythagoras, and your part (i) answer, determine the distance that I have walked.

Question 13

Two motor boats, *The Dragon*, and *The Runner*, sit side by side upon the ocean. They then separate, each at a constant velocity.

The Dragon has velocity $V_D = 4 i + 7 j \text{ kmh}^{-1}$

The Runner has velocity $V_{\mathbf{R}} = 5 \mathbf{i} + 5 \mathbf{j} \text{ kmh}^{-1}$

- (i) Which boat is faster and by how much?
- (ii) Calculate the velocity of *The Dragon* relative to *The Runner*.
- (iii) Use your part (ii) answer to determine how long it takes until the two motor boats are 5 km apart.

The velocities of particles A and B are $(u \, \boldsymbol{i} - 7 \, \boldsymbol{j}) \, \text{ms}^{-1}$ and $(5 \, \boldsymbol{i} + v \, \boldsymbol{j}) \, \text{ms}^{-1}$ respectively. The velocity of A relative to B is $(2 \, \boldsymbol{i} - 3 \, \boldsymbol{j}) \, \text{ms}^{-1}$ Find the values of u and v.

Question 15

The velocities of two particles A and B are (13 i - 3 j) ms⁻¹ and (5 i + 12 j) ms⁻¹ respectively.

Find;

- (\mathbf{i}) the speed of B,
- (ii) the velocity of B relative to A,
- (iii) the angle between this relative velocity and the positive *x*-axis direction, giving your answer to the nearest degree.

I am at the position r = 7 i + 5 j m. My velocity is given by v = 2 i + 4 j ms⁻¹

If I have no acceleration, what is my position 4 seconds later?

Question 17

The position of a particle at time t is given by $\mathbf{r} = (2t - 9)\mathbf{i} + (t - 2)\mathbf{j}$

(i) If d is the distance of r from the origin at time t, find an expression for d that involves the square root of a quadratic equation in t (Hint: Pythagoras).

(ii) Show, by completing the square on the quadratic, that;

$$\frac{1}{5}d^2 = (t-4)^2 + 1$$

- (iii) What value of t makes $\frac{1}{5}d^2$ as small as possible? This is the time at which the particle is closest to the origin.
- (iv) What is this minimum distance?