Chapter 4

A-Level Pure Mathematics
Vectors II : Year 1 and Year 2

4.1 Vectors: Topic Summary

Question 1

In a desert exercise a tank travels 12 km on a bearing of 070° from an Oasis, O, then 14 km on a bearing of 160° to a Bunker B.

(i) Provide a sketch of the tank's manoeuvres marking on the following;
12 km, 14 km, O, B, 70°, 90°

[2 marks]

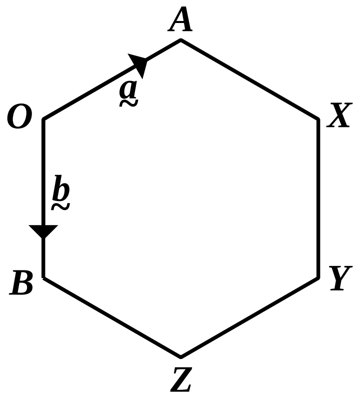
(ii) Determine the bearing of the tank's bunker location from the Oasis.

[2 marks]

Question 2

A regular hexagon has its six vertices marked O, A, X, Y, Z, and B as shown.

$$\overrightarrow{OA} = \boldsymbol{a} \quad \text{and} \quad \overrightarrow{OB} = \boldsymbol{b}$$



Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

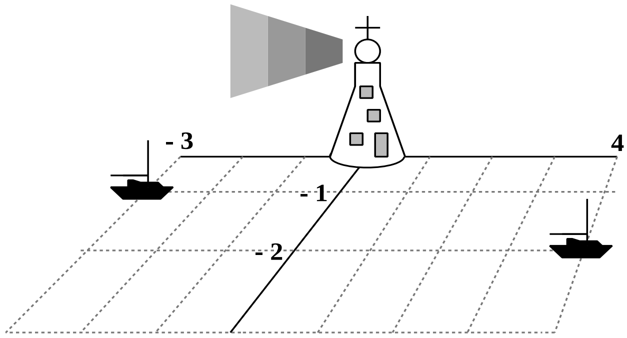
(i) $\overrightarrow{YX}$ **(ii)** $\overrightarrow{BX}$ **(iii)** $\overrightarrow{OZ}$

[3 marks]

Question 3

A yacht is initially at the position, $\boldsymbol{Y_A} = -3\,\boldsymbol{i} - \boldsymbol{j}$ km.

Some time later it is at position, $\boldsymbol{Y_B} = 4\,\boldsymbol{i} - 2\,\boldsymbol{j}$ km.



(i) Determine the vector that describes the change in position of the yacht.

[2 marks]

(ii) By using the theorem of Pythagoras, and your part (i) answer, determine the distance across the sea-bed that the yacht has covered.

[2 marks]

Question 4

Two motor boats, *The Chunter* and *The Rapid* sit side by side upon the ocean.
They then separate, each at a constant velocity.
The Chunter has velocity $\boldsymbol{V_C} = 3\,\boldsymbol{i} + 5\,\boldsymbol{j}$ kmh^{-1}
The Rapid has velocity $\boldsymbol{V_R} = 8\,\boldsymbol{i} + 4\,\boldsymbol{j}$ kmh^{-1}

(i) Calculate the speed of *The Chunter*.

[2 marks]

(ii) How far will *The Chunter* travel in 2 hours 15 minutes ?

[2 marks]

(iii) Calculate the velocity of *The Chunter* relative to *The Rapid*.

[2 marks]

(iv) Use your part (iii) answer to calculate, in hours and minutes, how long it will take until the two motor boats are 8 km apart.

[2 marks]

Question 5

A particle P has velocity ($3\,\boldsymbol{i} + 2\,\boldsymbol{j}$) ms^{-1} when $t = 0$ seconds
and velocity ($7\,\boldsymbol{i} + 4\,\boldsymbol{j}$) ms^{-1} at time $t = 2$ seconds
Find the acceleration of P assuming that it is constant.

[4 marks]

Question 6

The position of a particle at time t is given by; r = **(3*t* – 7)** i + **(6*t* + 1)** j

(i) If d is the distance in metres of ***r*** from the origin at time t, find an expression for d involving the square root of a quadratic equation in t (Hint: Pythagoras).

[2 marks]

(ii) Show, by completing the square on the quadratic, that;

$$\frac{1}{5}d^2 = 9\left(t - \frac{1}{3}\right)^2 + 9$$

[2 marks]

(iii) What value of t makes $\frac{1}{5}d^2$ as small as possible ?

This is the time at which the particle is closest to the origin.

[1 mark]

(iv) What is this minimum distance ?

[1 mark]

Question 7

At 11:00 hour the position vector of an aircraft relative to an airport *O* is;

$$\boldsymbol{r_A} = (200\, \boldsymbol{i} + 30\, \boldsymbol{j}) \text{ km}$$

Note that ***i*** and ***j*** are unit vectors due east and due north respectively.

The constant velocity of the aircraft is;

$$\boldsymbol{V_A} = (180\, \boldsymbol{i} - 120\, \boldsymbol{j}) \text{ kmh}^{-1}$$

Find;

(i) the time when the aircraft is due east of the airport *O*

[2 marks]

(ii) how far it then is from *O*

[2 marks]

(iii) how far it is from *O* at 12:00

[2 marks]