Number Theory I

	2	3		5		7			
11		13				17		19	
		23						29	
31						37			
41		43				47			
		53						59	
61						67			
71		73						79	
		83						89	
						97			

Number Theory I

Lesson 1

1.1 Primes

A prime number is a positive integer with two factors exactly.
A positive integer with more than two factors is said to be composite.
The number 1 is neither prime nor composite.
There are twenty-five primes less than 100.

	2	3		5		7			
11		13				17		19	
		23						29	
31						37			
41		43				47			
		53						59	
61						67			
71		73						79	
		83						89	
						97			

Primes are important because the composite numbers are built from primes.
Primes are the atoms of the mathematical universe.

1.2 The fundamental theorem of arithmetic

All composite numbers can be written as a product of primes.
The representation of any composite number in this way is essentially unique.

1.2.1 Examples

(i) $50 = 2 \times 5 \times 5$

(ii) $49 = 7 \times 7$

(iii) $19669 = 13 \times 17 \times 89$

1.3 Exercise

Write each of the following composite numbers as a product of prime numbers.

(i) 8 =

(ii) 10 =

(iii) 18 =

(iv) 50 =

(v) 30 =

(vi) 35 =

(vii) 70 =

(viii) 6 =

(ix) 100 =

(x) 121 =

(xi) 66 =

(xii) 52 =

(xiii) 98 =

1.3 B Extra Exercise

Write each of the following composite numbers as a product of prime numbers.

(i) 15 =

(ii) 14 =

(iii) 25 =

(iv) 27 =

(v) 24 =

(vi) 21 =

(vii) 110 =

(viii) 16 =

(ix) 200 =

(x) 44 =

(xi) 42 =

(xii) 90 =

(xiii) 91 =

1.4 Writing larger composites as products of primes.

There are two popular ways of doing this..
The cherry tree method or the division ladder method.

1.4.1 Example - the devil's number : 666

1.5 Exercise

Question 1
Use either the cherry tree method or the division ladder method to write each of the following composite numbers as a product of primes.

HINT : Keep an eye of the twenty-five primes less than 100.

(i) 564 **(ii)** 1550 **(iii)** 936

Question 2
Use either the cherry tree method or the division ladder method to write each of the following composite numbers as a product of primes.

(i) 1375 **(ii)** 6264

Question 3
In this question you can use a calculator.
Use either the cherry tree method or the division ladder method to write each of the following composite numbers as a product of primes.

(i) 213928 **(ii)** 1043504

Question 4

In this question you can use a calculator.

Use the number grid below to help work out which numbers between 101 and 200 are prime numbers.

HINT : There are 21 to be found.

101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

Question 5 : The Goldbach Conjecture

Every even number, other than 2, can be written as the sum of two primes.

This may or may not be true - it's a famous unresolved statement.

Examples : 4 = 2 + 2, 6 = 3 + 3, and 8 = 3 + 5.

Write each of the following even numbers as the sum of two primes

(i) 8 =

(ii) 20 =

(iii) 36 =

(iv) 70 =

(v) 98 =

(vi) 128 =

(vii) 162 =

(viii) 174 =

(ix) 196 =

(x) 200 =

Question 6 : Fermat's triangular triples

In 1636, Fermat claimed that

Every integer is the sum of three triangular numbers.

This was later proved to be true by Gauss.

In the following examples, note that 0 is considered to be a triangular number .

1 = 0 + 0 + 1					
2 = 0 + 1 + 1					
3 = 1 + 1 + 1	or	0 + 0 + 3			
4 =		0 + 1 + 3			
5 =		1 + 1 + 3			
6 =			0 + 3 + 3	or	0 + 0 + 6
7 =			1 + 3 + 3	or	0 + 1 + 6
8 =					1 + 1 + 6

Continue the above examples by writing each the numbers from 9 to 20 as the sum of three triangular numbers.

9 =

10 =

11 =

12 =

13 =

14 =

15 =

16 =

17 =

18 =

19 =

20 =

Question 7 : Sum of two squares primes

Another of Fermat's famous theorems says that :

If a prime has remainder 1 when divided by 4,
then that prime can be written as a sum of two squares.

Example

Consider the prime 41.
When divided by four this has remainder 1.
Thus 41 can be written as the sum of two squares.
After a bit of thought... $41 = 5^2 + 4^2$

Each of the following primes has remainder 1 when divided by 4.
Therefore they can be written as the sum of two squares.
In each case, find the two squares.

5 =

13 =

17 =

29 =

37 =

53 =

61 =

73 =

89 =

97 =