Lesson 2

Number Theory : Year 9

2.1 *hcf* and *lcm*

2.1.1 Example : *highest common factor*

The *highest common factor* of 6 and 15 is the biggest integer that divides into both 6 and 15 without remainder.

After a pause for thought : *hcf* {6,15} = 3.

2.1.2 Example : *lowest common multiple*

The *lowest common multiple* of 6 and 15 is the first integer that is in the 6 times table and also in the 15 times table.

After a pause for thought : *lcm* {6,15} = 30.

2.1.3 The connection between *hcf* and *lcm*

To work out an *lcm*, first find the *hcf*, then use;

$$lcm\{a,\ b\} \ = \ \frac{ab}{hcf\ \{a,\ b\}}$$

So, for example, to find *lcm* {18,33}.

First : Figure out that *hcf* {18,33} = 3.

Second : Work out $\frac{18 \ \times \ 33}{3}$ which is 198.

2.2 Exercise

Question 1

Write down the *hcf* and the *lcm* requested;

(i)	*hcf* {6, 10} =	*lcm* {6, 10} =
(ii)	*hcf* {2, 3}	*lcm* {2, 3} =
(iii)	*hcf* {12, 20} =	*lcm* {12, 20} =
(iv)	*hcf* {8, 16} =	*lcm* {8, 16} =
(v)	*hcf* {4, 9} =	*lcm* {4, 9} =
(vi)	*hcf* {26, 39} =	*lcm* {26, 39} =
(vii)	*hcf* {50, 80} =	*lcm* {50, 80} =
(viii)	*hcf* {30, 75} =	*lcm* {30, 75} =
(ix)	*hcf* {24, 36} =	*lcm* {24, 36} =
(x)	*hcf* {100, 105} =	*lcm* {100, 105} =
(xi)	*hcf* {8, 9} =	*lcm* {8, 9} =
(xii)	*hcf* {12, 16} =	*lcm* {12, 16} =
(xiii)	*hcf* {44, 66} =	*lcm* {44, 66} =

Question 2

Given that *hcf* {245, 350} = 35, use the following formula to find *lcm* {245, 350}.

$$lcm\{a,\ b\} \ = \ \frac{ab}{hcf\ \{a,\ b\}}$$

Question 3

I have a bag of sweets.
I could share it exactly between 15 people.
Or I could share it exactly between 27 people.

(i) What is *hcf* {15, 27} ?

(ii) What is *lcm* {15, 27} ?

(iii) What is the least number of sweets that must be in the bag ?

(iv) Will I definitely be able to share out the sweets exactly between 9 people ?

(v) Will I definitely be able to share out the sweets exactly between 45 people ?

Question 4

DEFINITION
Two numbers are coprime when the only factor they have in common is 1.
i.e. If *hcf* {a, b} = 1 then *a* and *b* are coprime.

(i) Are 18 and 75 coprime ?

(ii) Are 32 and 45 coprime ?

(iii) Are 49 and 63 coprime ?

Question 5
For each of the following pairs of integers, decide if they are coprime or not.

(i) 7 and 11

(ii) 15 and 21

(iii) 49 and 52

(iv) 10 and 11

(v) 13 and 39

(vi) 63 and 56

Question 6

(a)
Write 4000 as a product of primes.

(b)
Write 3969 as a product of primes.

(c) Are 4000 and 3969 coprime ?

(d) Find *lcm*{4000, 3969} giving the answer as a product of primes.

Question 7
Many mathematicians have searched for a formula that will generate some or all of the prime numbers. Unfortunately, although such formulae often initially seem to work well, a flaw seems to eventually always be found.

Here is one such formula.......

$$x^2 + x + 11 = prime$$

(i) Work out : $0^2 + 0 + 11 =$
Is the answer prime ?

(ii) Work out : $1^2 + 1 + 11 =$
Is the answer prime ?

(iii) Work out : $2^2 + 2 + 11 =$
Is the answer prime ?

(iv) Work out : $3^2 + 3 + 11 =$
Is the answer prime ?

(v) Work out : $4^2 + 4 + 11 =$
Is the answer prime ?

(vi) Work out : $5^2 + 5 + 11 =$
Is the answer prime ?

Now keep going. This formula is good enough to fool many people into thinking it will always give a prime number answer. Is it good enough to fool you ?

Question 8

Here is a 'mad' number tower;

$$4^{2^3} = ?$$

To understand it, calculate the 2^3 part first.
$2^3 = 2 \times 2 \times 2 = 8$
Then $4^8 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 65536$.
So,

$$4^{2^3} = 4^8 = 65536$$

Make sure that you can get your calculator to give this answer.

In 1640, Fermat thought he had discovered a formula for generating primes.
Here it is;

$$2^{2^x} + 1 = prime$$

(a) Find the numbers generated by this formula when;

(i) $x = 0$

(ii) $x = 1$

(iii) $x = 2$

(iv) $x = 3$

(v) $x = 4$

All the answers so far are prime.

(vi) $x = 5$

(b) This time the answer is not prime, although it would take a while to realise this as the smallest prime that divides into the answer is 641.

How many times does 641 divide into your part **(vi)** answer ?

Question 9

Here is a "prime number making machine".
It attempts to use all known primes to generate a new, previously unknown prime.

Take the first *x* primes.
Multiply them all together.
Add 1.
The answer is a new prime.

(i) Work out $2 \times 3 + 1$
Is it prime ?

(ii) Work out $2 \times 3 \times 5 + 1$
Is it prime ?

(iii) Work out $2 \times 3 \times 5 \times 7 + 1$
Is it prime ?

(iv) Work out $2 \times 3 \times 5 \times 7 \times 11 + 1$
Is it prime ?

(v) Work out $2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1$
This answer is NOT prime.
What divides into it ?