Lesson 3

Number Theory : Year 9

Calculator needed

3.1 Finding the *hcf* and *lcm* of pairs of large numbers

As the number pairs get bigger it becomes harder to mentally spot what the *hcf* and the *lcm* are. A system is needed to methodically work them out.

3.2 Strategy

To find the *hcf* and *lcm* of a and b.

- ◇ **STEP 1 :** Write each of a and b as a products of primes.
- ◇ **STEP 2 :** The *hcf* $\{a, b\}$ is all the **common** primes multiplied together.
- ◇ **STEP 3 :** The *lcm* $\{a, b\}$ is $\frac{a\,b}{hcf\{a,b\}}$

3.3 Example

Find the *hcf* and *lcm* of 210 and 245.

210	=	$2 \times 3 \times 5 \times 7$	Use calculator's FACT button
245	=	$5 \times 7 \times 7$	Use calculator's FACT button
hcf	=	5×7	

$\therefore$ *hcf* $\{210, 245\} = 35$

$$lcm = \frac{210 \times 245}{35}$$

$$= 1470$$

$\therefore$ *lcm* $\{210, 245\} = 1470$

(*Using a calculator*)

Three cheers for the FACT button !

FACT = "What are the prime factors of ? "

Mavis : "I had a really big number† to break down into prime factors.
I was in tears but then George showed me the FACT button.
It saved my life."

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† Most numbers are big

3.4 Exercise

Question 1
Find the *hcf* and *lcm* of 275 and 297

Question 2
Find;

(i) *hcf* {220, 462}
(ii) *lcm* {220, 462}

Question 3

(i) Write 336 as a product of primes

(ii) Write 630 as a product of primes

(iii) Calculate *hcf* {336, 630}

(iv) Calculate *lcm* {336, 630}

Question 4

(i) Find the *lcm* of 9 and 6
(HINT : Work out the *hcf* first)

(ii) Find the *lcm* of 20 and 12

(iii) Find the *lcm* of 18 and 12

(iv) Find the *lcm* of 24 and 18

Question 5

(i) Write 875 as a product of primes

(ii) Write 910 as a product of primes

(iii) Calculate *hcf* {875, 910}

(iv) Calculate *lcm* {875, 910}

Question 6

(i) Find the *lcm* of 7 and 11
(Hint : Work out the *hcf* first)

(ii) Find the *lcm* of 5 and 13

(iii) Two prime numbers are p and q
What is *lcm* $\{p, q\}$?

Question 7

(i) Find the *lcm* of 4 and 16

(ii) Find the *lcm* of 5 and 25

(iii) Two numbers are s and s^2
What is *lcm* $\{s, s^2\}$?

Question 8

(i) Find the *lcm* of 2 and 3

(ii) Find the *lcm* of 3 and 4

(iii) Two consecutive integers are a and b
What is the *lcm* $\{a, b\}$?

Question 9
Find the *lcm* of 525 and 2205

Question 10

A Table of Multiples

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
2	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**	**22**	**24**	**26**	**28**	**30**	**32**	**34**	**36**
3	**6**	**9**	**12**	**15**	**18**	**21**	**24**	**27**	**30**	**33**	**36**	**39**	**42**	**45**	**48**	**51**	**54**
4	**8**	**12**	**16**	**20**	**24**	**28**	**32**	**36**	**40**	**44**	**48**	**52**	**56**	**60**	**64**	**68**	**72**
5	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**
6	**12**	**18**	**24**	**30**	**36**	**42**	**48**	**54**	**60**	**66**	**72**	**78**	**84**	**90**	**96**	**102**	**108**
7	**14**	**21**	**28**	**35**	**42**	**49**	**56**	**63**	**70**	**77**	**84**	**91**	**98**	**105**	**112**	**119**	**126**
8	**16**	**24**	**32**	**40**	**48**	**56**	**64**	**72**	**80**	**88**	**96**	**104**	**112**	**120**	**128**	**136**	**144**
9	**18**	**27**	**36**	**45**	**54**	**63**	**72**	**81**	**90**	**99**	**108**	**117**	**126**	**135**	**144**	**153**	**162**
10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**	**140**	**150**	**160**	**170**	**180**
11	**22**	**33**	**44**	**55**	**66**	**77**	**88**	**99**	**110**	**121**	**132**	**143**	**154**	**165**	**176**	**187**	**198**
12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**132**	**144**	**156**	**168**	**180**	**192**	**204**	**216**

Using the table of multiples, or otherwise, write down;

(i) *lcm*{2, 5}

(ii) *lcm* {3, 7}

(iii) *lcm* {8, 12}

(iv) *lcm* {4, 9}

Which of **(i), (ii), (iii)** and **(iv)** disproves the following theories;

THEORY A : The *lcm* of two numbers, *x* and *y*, is **always** *x* times *y*.

THEORY B : The *lcm* of two numbers, *x* and *y*, is *x* times *y* **only** if *x* and *y* are prime numbers.

Write down a correct theory that tells you when the *lcm* of two numbers, *x* and *y* is *x* times *y*.