3.1 Trigonometric Formulae

Many useful formulae include the trigonometric functions *sin x*, *cos x* or *tan x*.
This Lesson is about extracting useful information from such formulae.

3.2 Example

In the UK, domestic mains electricity is supplied as alternating current.
This has maximum value of 240 volts and minimum voltage of – 240 volts.
It comes as a fairly perfect sine wave at a frequency of 50 Hz.

As a mathematical formula; $V = 240\,\sin(18000\,t + \alpha)$

where V is the voltage (volts) at time, t (seconds).
α is one of three phases, either $\alpha = 0°$, or $\alpha = 120°$ or $\alpha = 240°$.
If your house is on $\alpha = 120°$ then your neighbours house on one side will be on $\alpha = 0°$ and your neighbours house on the other side will be on $\alpha = 240°$.
Find the voltage when

(i) $t = 0.0025$ seconds, and $\alpha = 120°$

(ii) $t = 0.005$ seconds, and $\alpha = 240°$

3.3 Exercise

Question 1

In the harbour at Holyhead, Wales, the height of the tide on a certain day is given by;

$$H = 3.1 + 2.9\,\sin(28.8t + 40)$$

where H is the height of the tide in metres at time t in hours after midnight.
Find the height of the tide in Holyhead harbour at

(i) $t = 1$ am

(ii) $t = 2$ am

(iii) $t = 3$ am

(iv) One of these is high tide. Was high tide at 1am, 2am or 3am ?

(v) Experiment to find the time, to the nearest hour, of low tide.

Question 2

In the drying harbour at Paignton, Devon, the height of the tide on a certain day is given by;

$$H = 2.4 + 2.7\,cos\,(28.8t - 74)$$

where H is the height in hours and t is the time in hours after midnight.

Find the height of the tide in Paignton harbour at

(i) t = 17:00

(ii) t = 18:00

(iii) t = 19:00

(iv) If the harbour dries when the tide is below 1.4 m, did the harbour become dry at 17:00, 18:00 or 19:00 ?

(v) Experiment to find the time, to the nearest hour, at which the harbour will start to refill with water.
(i.e. When there is next more than 1.4 m of tide)

Question 3

A boy, bouncing on a pogo stick, has eyes at a height, H, above the ground given by;

$$H = 2.3 + 0.5\,cos\,(120t + 45)$$

where t is the time in seconds after he first started bouncing.

How high are his eyes above the ground when

(i) t = 6 seconds,

(ii) t = 6.5 seconds

(iii) t = 7 seconds

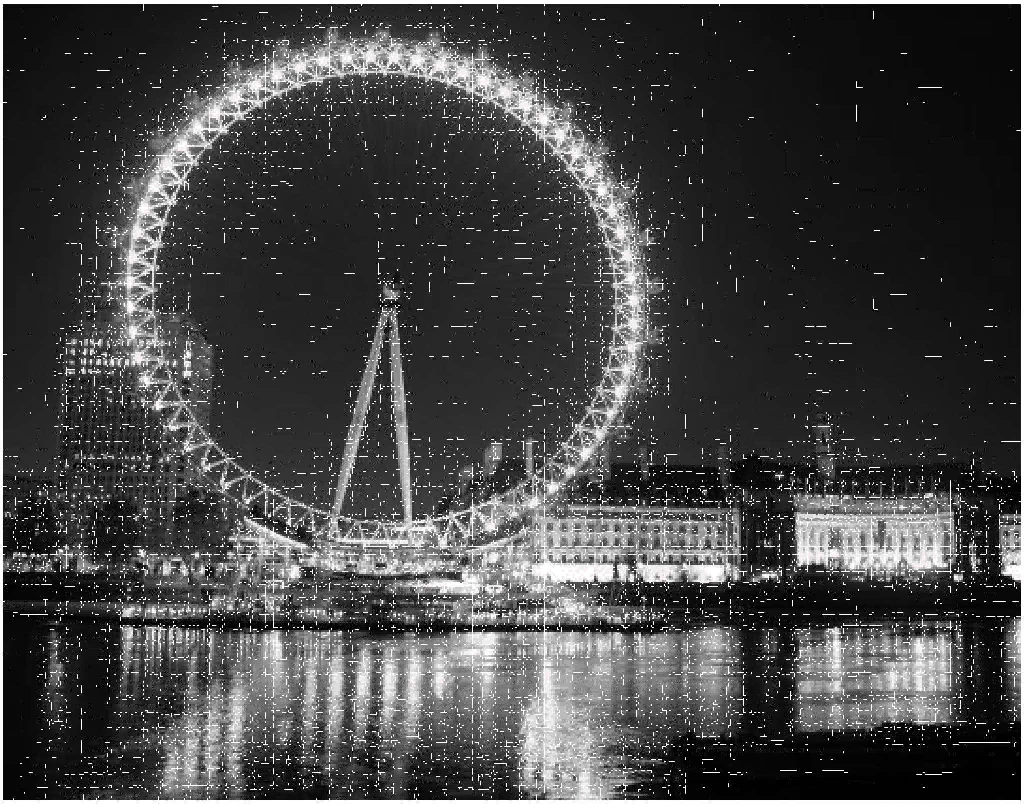
(iv) Experiment to find the first time after 7 seconds, to the nearest half second, when he gets to his maximum height.

Question 4

The photograph below is of the London Eye, a large ferris wheel.

The height above the ground, *H*, of a passenger, *t* minutes after stepping aboard is;

$$H = 170 + 130\,sin\,(12t - 90)$$



How high is a passenger at the following times after stepping aboard;

(i) $t = 3$ minutes

(ii) $t = 6$ minutes

(iii) $t = 9$ minutes

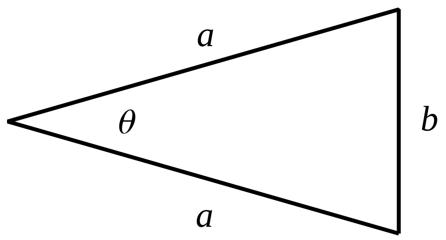
(iv) Experiment to find the time, to the nearest three minutes, for the passenger to get to the maximum height.

(v) How long does it take the wheel to revolve once ?

Question 5

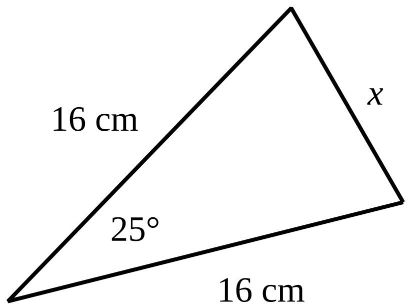
Given an isosceles triangle with two sides of length a, at an angle of θ apart, the length of the third side, b, is given by the formula

$$b^2 = 2a^2(1 - \cos\theta)$$

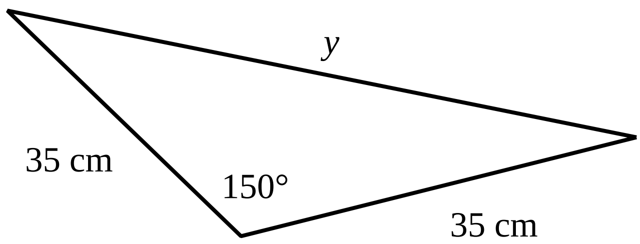


Use the formulae to find the length of the missing side of each of these triangles; Give your answers to three significant figures.

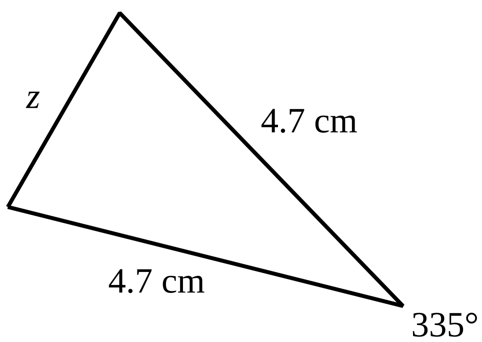
(i)



(ii)



(iii)

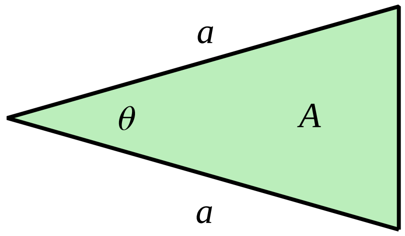


Can the formulae cope with having 335° inserted or do you have to use 360° – 335° instead ?

Question 6

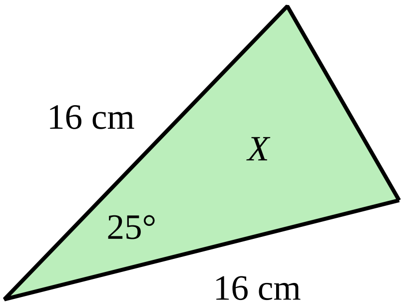
Given an isosceles triangle with two sides of length *a*, at an angle of θ apart, the area, *A*, is given by the formula

$$A = \frac{1}{2} a^2 \sin\theta$$

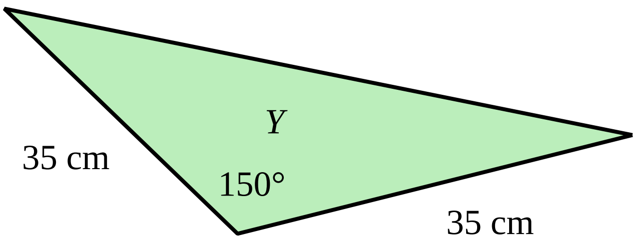


Use the formulae to find the area of each of these triangles;
Give your answers to 3 significant figures.

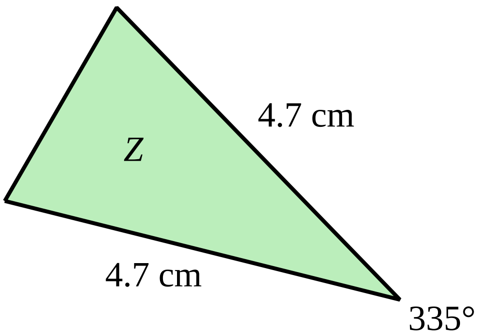
(i)



(ii)



(iii)



Can the formulae cope with having 335° inserted or do you have to use 360° – 335° instead ?

Question 7

The area of any regular polygon is given by the formula;

$$A = \frac{n\,a^2}{4\,tan\left(\frac{180}{n}\right)}$$

where n is the number of sides,
and a is the length of a side.

(i) A square of side 8 cm has an area of 64 cm^2
Show clearly that the formula also gives the area of the square to be 64 cm^2

(ii) Use the formula to find the area of a regular decagon of side length 6.2 cm.

(iii) Use the formula to find the area of a regular hexagon of side length 17.2 cm.

(iv) Find the area of a regular 100 sided polygon of with a perimeter of 80 cm.

Image : Millennium Wheel, London by Dreamer