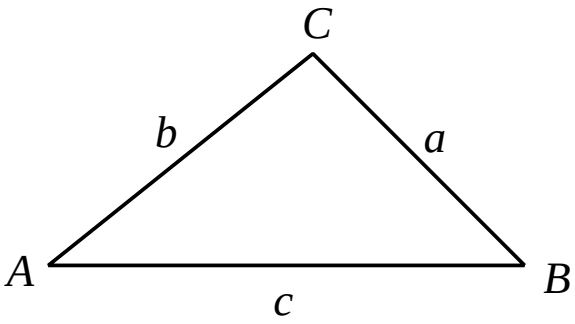
Lesson 6

GCSE Mathematics

Non-Right Angled Trigonometry

6.1 Elur Enisoc : The Cosine Rule Reversed

Here is The Cosine Rule, as given on the GCSE examination formula page;



$$a^2 = b^2 + c^2 - 2bc\cos A$$

Here is how to make *cos A* the subject of this formula.

Step 1 : Add $2bc\cos A$ to both sides...

$$2bc\cos A + a^2 = b^2 + c^2$$

Step 2 : Subtract a^2 from both sides...

$$2bc\cos A = b^2 + c^2 - a^2$$

Step 3 : Divide both sides by $2bc$...

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

We shall refer to this result as The Cosine Rule Reversed : "Elur Enisoc".
It is not given on the GCSE examination page so you need to
Either

- Remember it, or rather the form (look at where *A* and *a* are).

Or

- Derive it from the version given in the examination, as shown above.

TOP TIP : When entering this into your calculator, use the fraction button.
If your calculator does not have a fraction button put brackets in, like this...

$$\cos A = \frac{\left(b^2 + c^2 - a^2\right)}{\left(2bc\right)}$$

Use The Cosine Rule Reversed when you know all three sides and want to find an angle

6.2 Exercise

Question 1
For $\triangle FUN$, write down the reversed cosine rule for $\cos F$, in terms of f, u and n.

Question 2
For $\triangle HAT$, write down the reversed cosine rule for $\cos T$, in terms of h, a and t.

Question 3
For $\triangle DAZ$, write down the reversed cosine rule for $\cos Z$.

Question 4
For $\triangle ABC$, write down the reversed cosine rule for;

(i) $\cos C$

(ii) $\cos B$

(iii) $\cos A$

Question 5

(i) Solve the equation $X = \arccos 0.822$
Give your answer to 1 decimal place.

(ii) Solve the equation $Y = \arccos(-0.109)$
Give your answer to 1 decimal place.

(iii) If $\cos Z = 0.571$ what is Z ?
Give your answer in degrees, to 1 decimal place.

(iv) If $\cos E = 0.311$ what is E ?
Give your answer in degrees, to 1 decimal place.

(v) If $\cos W = -0.287$ what is W ?
Give your answer in degrees, to 1 decimal place.

Question 6

A triangle has sides of length 3 cm, 7 cm and 9 cm.

(i) Roughly sketch the triangle, with the sides roughly the correct lengths.

(ii) Is the largest angle

- Opposite the 3 cm side ?

Or

- Opposite the 7 cm side ?
- Opposite the 9 cm side ?

Question 7

In $\triangle ABC$, $BC = 13$ m, $CA = 8$ m and $AB = 14$ m.

(i) Sketch, roughly, $\triangle ABC$ marking on the three known lengths.

(ii) Write down the formula for the reversed cosine rule for *cos* A.

(iii) Find the size of angle *A*.

Question 8

In $\triangle ABC$, $BC = 8$ cm, $CA = 12$ cm and $AB = 9$ cm.

(i) Sketch, roughly, $\triangle ABC$ marking on the three known lengths.

(ii) Write down the reversed cosine rule for *cos* A.

(iii) Find the size of angle *A*.

Question 9

In $\triangle YXU$, $XU = 280$ m, $UY = 240$ m and $YX = 200$ m.

(i) Sketch, roughly, $\triangle YXU$ marking on the three known lengths.

(ii) Write down the reversed cosine rule for $\cos Y$.

(iii) Find the size of angle Y, giving your answer in degrees to one decimal place.

Question 10

In $\triangle QPR$, $PR = 51\,400$ km, $RQ = 72\,300$ km and $QP = 84\,400$ km.

(i) Sketch, roughly, $\triangle QPR$ marking on the three known lengths.

(ii) Write down the reversed cosine rule for $\cos R$.

(iii) Find the size of angle R, giving your answer in degrees to one decimal place.

Question 11

The AREA of a triangle is given on the GCSE examination formula page as

$$Area\ of\ triangle\ =\ \frac{1}{2}\,ab\,\sin C$$

(i) Make *sin C* the subject of this formula.

(ii) A triangle has an area of 52 cm^2
It has one side of length 18 cm, and another side of length 11 cm.
What is the size of the acute angle between these two sides ?
Give your answer in degrees to one decimal place.
HINT : Your **part (i)** formula should help !

(iii) If you knew that the angle was obtuse, what would it be ?

Question 12

In $\triangle TVC$, $VC = 11$ cm, $CT = 8$ cm and $TV = 10$ cm.

(i) Sketch, roughly, $\triangle TVC$ marking on the three known lengths.

(ii) Write down the reversed cosine rule for $\cos T$.

(iii) Find the size of angle T.

(iv) Write down the reversed cosine rule for $\cos V$.

(v) Find the size of angle V.

(vi) Using a well known fact about the sum of the angles in a triangle, deduce the size of the remaining angle, angle C.