

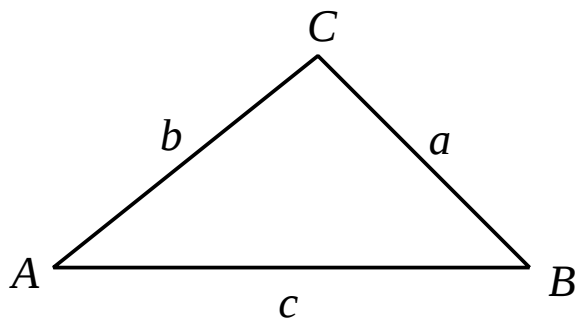
Lesson 7

GCSE Mathematics Non-Right Angled Trigonometry

7.1 The Sine Rule

On the GCSE examination formulae page...

In any triangle ABC



$$\text{Sine Rule : } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine Rule : } a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$

The lower two of these rules we recognise; it is the upper one we now consider.
(Notice that The Cosine Rule Reversed is not given)

The Sine Rule may look somewhat odd at first glance as it contains two equals signs.
It's a "three in one" formula !

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Tells us that:

(i)

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

(ii)

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

(iii)

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

***Use it when you know one side and two (and $\therefore$ three) angles
and want to find a side***

7.2 Example

The Question

$\triangle XTC$ has $\angle X = 54^\circ$ and $\angle T = 80^\circ$.

Side XT is of length 8.3 cm.

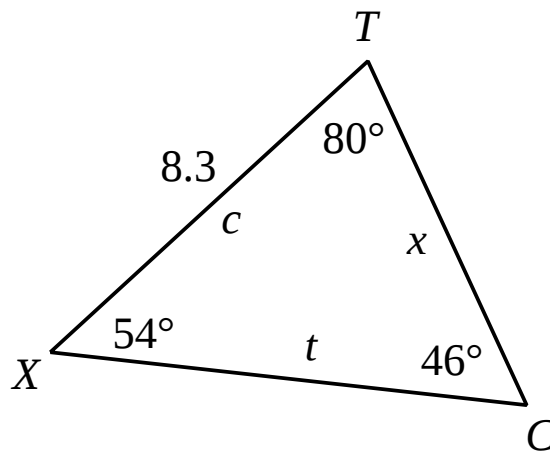
- (i) What is the size of angle C , the third angle ?
- (ii) Sketch the triangle, not to scale, and mark on all known lengths and angles.
- (iii) Find the length of side TC correct to three significant figures.

The Solution

(i)

$$\begin{aligned}\angle C &= 180 - (54 + 80) \\ &= 46^\circ\end{aligned}$$

(ii)



(iii) To find TC , or x ;

- ☐ Write out The Sine Rule in full for $\triangle XTC$
- ☐ Circle which to find
- ☐ Box known pair

$$\frac{\textcircled{x}}{\sin X} = \frac{t}{\sin T} = \boxed{\frac{c}{\sin C}}$$

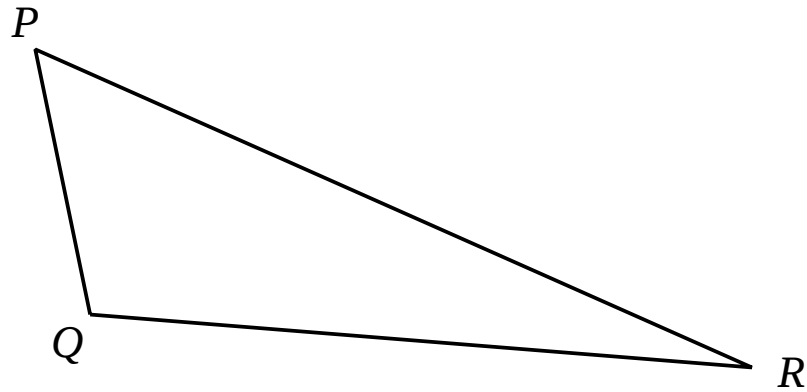
$$\begin{aligned}\frac{x}{\sin X} &= \frac{c}{\sin C} \\ \frac{x}{\sin 54} &= \frac{8.3}{\sin 46} \\ x &= \frac{8.3 \times \sin 54}{\sin 46}\end{aligned}$$

$$x = 9.33 \text{ cm} \quad \text{to 3 significant figures}$$

7.3 Exercise

Question 1

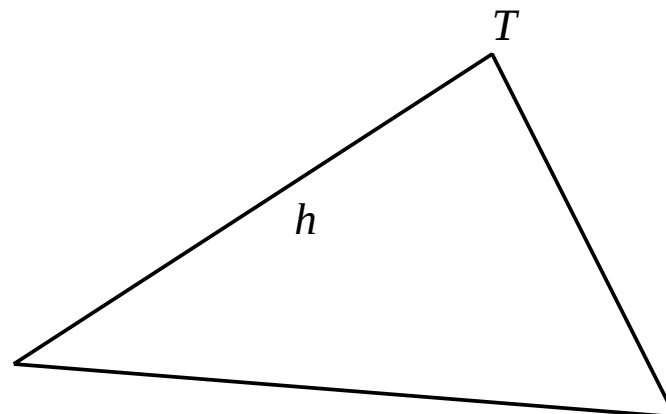
- (i) On the following diagram place the letters p , q and r on the sides opposite the similarly labelled angles, P , Q and R .



- (ii) Write down the sine rule in terms of P , p , Q , q , R and r .

Question 2

- (i) On the following diagram place the letters H , t , V and v .



- (ii) Write down the sine rule in terms of H , h , T , t , V and v .

Question 3

In each case, calculate the value of x correct to three significant figures;

(i)

$$x = \frac{246.4 \times 0.5643}{0.8025}$$

(ii)

$$x = \frac{5.8 \times \sin 66^\circ}{\sin 23^\circ}$$

(iii)

$$x = \frac{0.71 \times \sin 122^\circ}{\sin 31^\circ}$$

(iv)

$$x = \frac{2461 \times \sin 62^\circ}{\sin 34^\circ}$$

Question 4

Solve the following equations to find the values of x correct to three significant figures.

(i)

$$\frac{x}{8} = \frac{5}{7}$$

(ii)

$$\frac{x}{\sin 68^\circ} = \frac{74}{\sin 18^\circ}$$

(iii)

$$\frac{x}{\sin 106^\circ} = \frac{2375}{\sin 76^\circ}$$

(iv)

$$\frac{x}{\sin 60} = \frac{63}{\sin 90^\circ}$$

Question 5

$\triangle ABC$ has $\angle A = 28^\circ$, and $\angle B = 74^\circ$.

Side AC is of length 51 cm.

- (i) What is the size of angle C , the third angle ?
- (ii) Sketch the triangle, not to scale, and mark on all known lengths and angles.
- (iii) Find the length of side BC correct to three significant figures.

HINT ☐ Write out The Sine Rule in full for $\triangle ABC$

☐ Circle which to find

☐ Box known pair

- (iv) Find the area of $\triangle ABC$ by using the following formula;

$$\text{Area } \triangle = \frac{1}{2} a b \sin C$$

Question 6

$\triangle DEF$ has $\angle E = 18^\circ$ and $\angle F = 110^\circ$.

Side EF is of length 6.4 cm.

- (i) What is the size of angle D , the third angle ?
- (ii) Sketch the triangle, not to scale, and mark on all known lengths and angles.
- (iii) Find the length of side DF correct to three significant figures.

HINT ☐ Write out The Sine Rule in full for $\triangle DEF$

☐ Circle which to find

☐ Box known pair

- (iv) Dr Goodsum has worked out that the triangle's area is given by;

$$Area \triangle = \frac{1}{2} d e \sin F$$

Calculate the area of $\triangle DEF$.

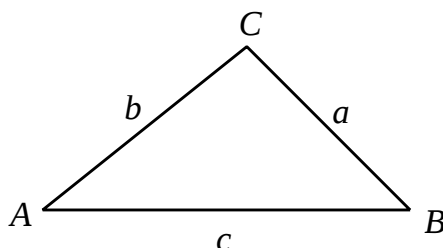
Question 7

$\triangle PDQ$ has $\angle P = 44^\circ$ and $\angle D = 69^\circ$.

Side PD is of length 18 cm.

- (i) What is the size of angle Q , the third angle ?
- (ii) Sketch the triangle, not to scale, and mark on all known lengths and angles.
- (iii) Find the length of side DQ correct to three significant figures.
- (iv) Find the length of side PQ correct to three significant figures.
- (v) The following formula gives the area of a triangle when you don't know the triangle's perpendicular height;

$$\text{Area } \triangle = \frac{1}{2} a b \sin C$$



Use an appropriate version of the area formula to determine area of $\triangle PDQ$.