Lesson 8

GCSE Mathematics

Non-Right Angled Trigonometry

8.1 The Upside Down Sine Rule

Previously, The Sine Rule was used to find the length of a triangle's side given

- another length

and

- two (and ∴ three) angles.

If we know

- the length of two of a triangle's sides

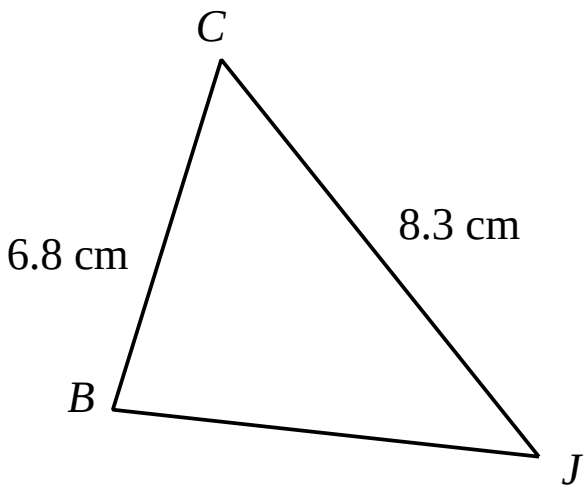
and

- an excluded angle

then The Sine Rule can be used 'upside down' to find the other excluded angle.

Example 1

(i) On the following triangle mark the two excluded angles, each with a *.



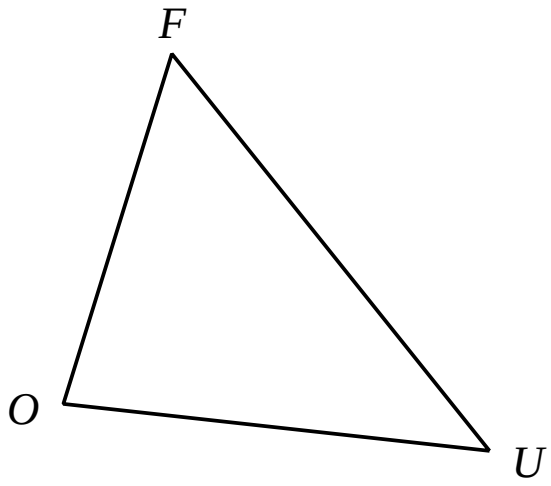
(ii) On the diagram place the letters *j*, *c* and *b*.

(iii) Write down The Sine Rule in terms of *J*, *j*, *C*, *c*, *B* and *b*.

(iv) Write down the corresponding 'upside down' version of The Sine Rule.

Example 2

A triangle, ΔUFO, is shown below.



(i) On the triangle place the letters *u*, *f* and *o*.

(ii) Onto the triangle add the facts that

- $\angle U = 53°$
- $UF = 8.4$ cm
- $FO = 7.6$ cm
- $\angle O = x$, the angle to be found.

(iii) On the triangle mark the two excluded angles, each with a *.

(iv) Write down the 'upside down' version of The Sine Rule for the triangle.

(v) Find angle *x*, in degrees and accurate to 3 significant figures.

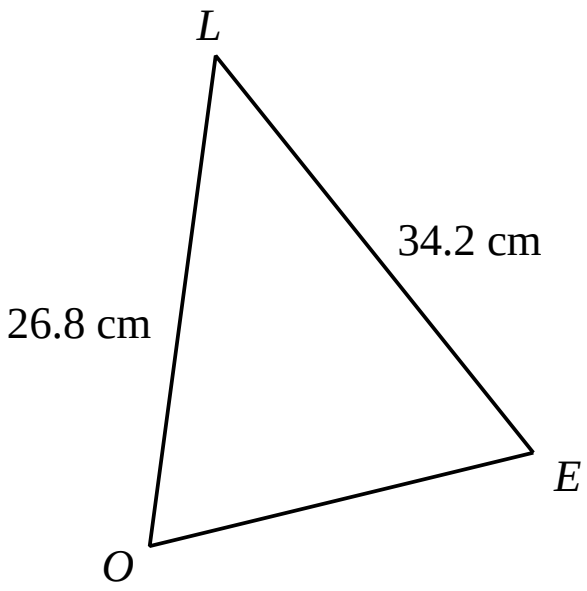
HINT ☐ Circle which to find

☐ Box known pair

8.2 Exercise

Question 1

(i) On the following triangle mark the two excluded angles, each with a *.



(ii) On the diagram place the letters *e, l* and *o.*

(iii) Write down The Sine Rule in terms of *E, e, L, l, O* and *o*.

(iv) Write down the corresponding 'upside down' version of the sine rule.

Question 2

Which is the correct ending for the given sentence ?

I'd use The Sine Rule (upside down version) to find an excluded angle if I knew....

(a) ☐ The lengths of all three sides of a triangle, and no angles.

(b) ☐ The length of two sides and the included angle.

(c) ☐ The length of two sides and one excluded angle.

(d) ☐ All three angles, but not the length of any side.

Question 3

In each case, calculate the value of x in degrees, correct to three significant figures;

(i) $\sin x = 0.334$

(ii) $\sin x = \frac{67}{80}$

(iii)

$$\sin x = \frac{41.6 \times 0.5643}{34.2}$$

(iv)

$$\sin x = \frac{15.8 \times \sin 42^\circ}{21.5}$$

(v)

$$\sin x = \frac{7.1 \times \sin 62^\circ}{11.8}$$

(vi)

$$\sin x = \frac{28.6 \times \sin 82^\circ}{45}$$

Question 4

Solve to find the values of x in degrees, correct to three significant figures.

(i)

$$\frac{\sin x}{5} = \frac{9}{62}$$

(ii)

$$\frac{\sin x}{55} = \frac{\sin 24^\circ}{73}$$

(iii)

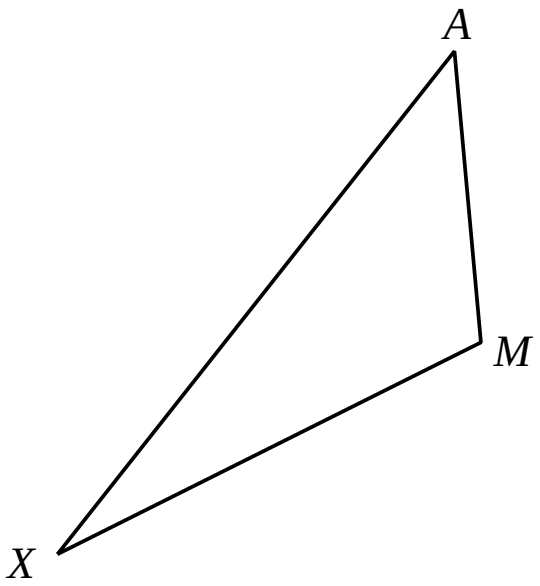
$$\frac{\sin x}{16} = \frac{\sin 35^\circ}{23}$$

(iv)

$$\frac{\sin x}{5.4} = \frac{\sin 60}{4.9}$$

Question 5

A triangle, $\triangle MAX$, is shown below.



(i) On the triangle place the letters *m*, *a* and *x*.

(ii) Onto the triangle add the facts that

- $\angle M = 104°$
- $MX = 7.8$ cm
- $XA = 12.7$ cm
- $\angle A = x$, the angle to be found.

(iii) On the triangle mark the two excluded angles, each with a *.

(iv) Write down the 'upside down' version of The Sine Rule for the triangle.

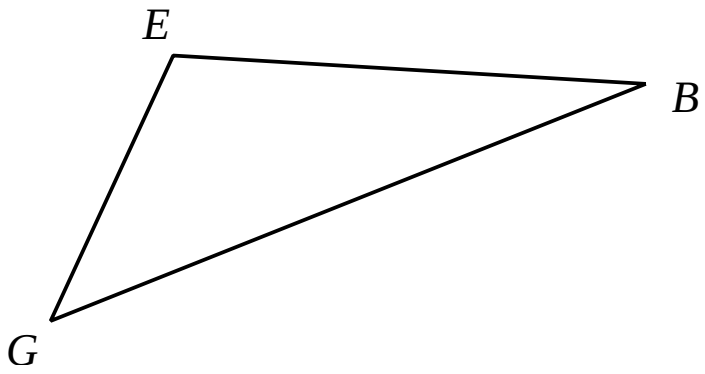
(v) Find angle *x*, in degrees and accurate to 3 significant figures.

HINT ☐ Circle which to find

☐ Box known pair

Question 6

A triangle, $\triangle BEG$, is shown below.



(i) On $\triangle BEG$ place the letters b, e and g.

(ii) On $\triangle BEG$ add the facts that

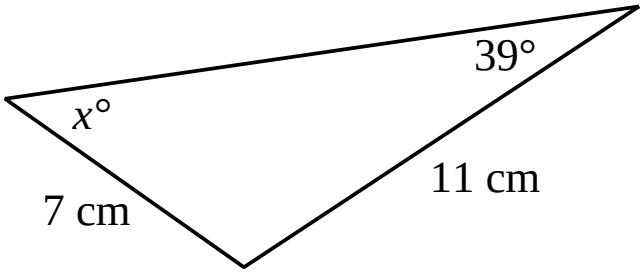
- $\angle B = x°$, the angle to be found.
- $BG = 17.8$ cm
- $GE = 14.9$ cm
- $\angle E = 130°$.

(iii) On $\triangle BEG$ mark the two excluded angles, each with a *.

(iv) Write down the 'upside down' version of the sine rule for $\triangle BEG$.

(v) Find angle x, in degrees and accurate to 3 significant figures.

Question 7



Find the size of the angle marked *x*, in degrees and accurate to 3 significant figures.

Question 8

This is a tough question.

$\triangle XYZ$ has XY of length 13.2 cm and length XZ of length 27.5 cm.

The angle at vertex Z is 24°.

Find the included angle at vertex X.

HINT : Find the excluded angle at vertex Y first, as this can be obtained using the 'upside down' version of The Sine Rule.