Lesson 3

Additional Mathematics
A-Level Pure Mathematics : Year 1
Topics In Algebra

3.1 Completing the square

Example
Solve this equation by completing the square, giving an exact answer.

$$x^2 - 6x = 2$$

[2 marks]

3.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 70

Question 1
Solve these equations by completing the square, giving exact answers.

(i) $x^2 - 8x = 1$ **(ii)** $x^2 + 2x = 5$

(iii) $x^2 - 12x = 5$ **(iv)** $x^2 + 14x + 30 = 0$

[8 marks]

Question 2

By completing the square on $x^2 + 6x = 11$ show that the equation has solutions of the form $a \pm b\sqrt{c}$ where a, b and c are integers and c is square free.

[3 marks]

Question 3

Solve these equations by completing the square, giving exact answers.

(i) $(x - 5)(x + 3) = 1$

(ii) $x + \frac{1}{x} = 4$

(iii) $x(x + 4) = 7$

(iv) $\frac{1}{(x + 1)} + x = 3$

[12 marks]

Question 4

The following equation is to be solved by the method of completing the square;

$$\frac{2}{x} + \frac{17}{x^2} = 1$$

Show that the exact solutions are of the form $x = a + b\sqrt{c}$
where a, b and c are integers and c is square free.

[4 marks]

Question 5

The following equation is to be solved by the method of completing the square;

$$\frac{5}{2x} = \frac{x - 20}{4}$$

Show that the exact solutions are of the form $x = a + b\sqrt{c}$
where a, b and c are integers and c is square free.

[4 marks]

Question 6

Examination question from May 2005, Q3

$$x^2 - 8x - 29 \equiv (x + a)^2 + b$$

where a and b are constants.

(a) Find the value of a and the value of b.

[2 marks]

(b) Hence, or otherwise, show that the roots of

$$x^2 - 8x - 29 = 0$$

are $c \pm d\sqrt{5}$ where c and d are integers to be found.

[3 marks]

Question 7

$$x^2 - 3x + 1 \equiv (x + a)^2 + b$$

where a and b are rational constants.

(a) Find the value of a and the value of b.

[2 marks]

(b) Hence, or otherwise, show that the roots of

$$x^2 - 3x + 1 = 0$$

are $c \pm d\sqrt{5}$ where c and d are rational constants to be found.

[3 marks]

Question 8

Find, as surds, the roots of the equation:

$$(x - 2)^2 = 2(x + 1)(x - 4)$$

[4 marks]

Question 9

The following equation is to be solved by the method of completing the square;

$$x^2 - 3x - 1 = 0$$

Show that the exact solutions are of the form;

$$x = \frac{a \pm \sqrt{b}}{2}$$

clearly stating the values of the integers, a and b.

[3 marks]

Question 10

Use the method of completing the square to solve the equation;

$$x - 2x^{\frac{1}{2}} - 1 = 0$$

Give your answers in the exact form;

$$x = a + b\sqrt{c}$$

where a, b and c are integers and c is square free.

[4 marks]

Question 11

Peter is investigating quadratic equations of the form;

$$x^2 + 2px + p = 0$$

where p is a constant and x is a variable.

(i) By completing the square, find the algebraic solution to all equations of this type.

[3 marks]

(ii) Use your part (i) answer to write down the equation that corresponds to $p = 10$, and its solution.

[1 mark]

(iii) Use your part (i) answer to write down the equation that corresponds to $p = 12$, and its solution.

[1 mark]

(iv) Use your part (i) answer to write down the equation that corresponds to $p = 0.5$
Explain why this equation has no solutions.

[2 marks]

(v) What relationship must hold for one of Peter's equations to have solutions ?

[2 marks]

Question 12

Peter's friend, Sophia, is investigating quadratic equations of the form;

$$x^2 + 2px + q = 0$$

where p and q are constants and x is a variable.

(i) By completing the square, find the algebraic solution to all equations of this type.

[3 marks]

(ii) Use your part (i) answer to write down the equation that corresponds to $p = 3$, $q = 5$, and its solution.

[1 mark]

(iii) Use your part (i) answer to write down the equation that corresponds to $p = -4$, $q = 11$, and its solution.

[1 mark]

(iv) Use your part **(i)** answer to write down the equation that corresponds to $p = 3$, $q = 10$
Explain why this equation has no solutions.

[2 marks]

(v) What relationship must hold for one of Sophia's equations to have solutions ?

[2 marks]