Lesson 4

Additional Mathematics
A-Level Pure Mathematics : Year 1
Topics In Algebra

4.1 Completing The Square : Tougher Problems

4.1.1 Example

By completing the square, solve the equation :

$$2x^2 + 3x - 3 = 0$$

Give your answer in the form $a \pm b\sqrt{33}$ for some constants a and b.

4.1.2 Model Solution :

Pull out the coefficient of x^2 from the terms in x^2 and x :

$$2\left[x^2 + \frac{3}{2}x \right] - 3 = 0$$

Complete the square within the square brackets :

$$2\left[\left(x + \frac{3}{4} \right)^2 - \frac{9}{16} \right] - 3 = 0$$

Expand the square brackets :

$$2\left(x + \frac{3}{4} \right)^2 - \frac{9}{8} - 3 = 0$$

Tidy up :

$$2\left(x + \frac{3}{4} \right)^2 - \frac{9}{8} - \frac{24}{8} = 0$$

$$2\left(x + \frac{3}{4} \right)^2 - \frac{33}{8} = 0$$

$$\left(x + \frac{3}{4} \right)^2 = \frac{33}{16}$$

Square root both sides :

$$x + \frac{3}{4} = \pm\sqrt{\frac{33}{16}}$$

$$x = -\frac{3}{4} \pm \frac{\sqrt{33}}{4}$$

[4 marks]

4.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 36

Question 1

Without using a calculator, use the method of completing the square to solve these equations;

(i) $2x^2 + 7x + 1 = 0$

[4 marks]

(ii) $5x^2 + 7x + 1 = 0$

[4 marks]

(iii) $3x^2 + 5x + 1 = 0$

[4 marks]

(iv) $10x^2 + 3x - 2 = 0$

[4 marks]

Question 2

Without using a calculator, by completing the square, solve the equation;

$5x^2 + 4x - 2 = 0$

[4 marks]

Question 3

Without using a calculator, by completing the square, show that the solutions to the equation; $2x^2 - 12x + 17 = 0$ are;

$$x = 3 \pm \frac{\sqrt{2}}{2}$$

[4 marks]

Question 4

Without using a calculator, by completing the square, show that the solutions to the equation; $2x^2 + 2x - 1 = 0$ are;

$$x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$$

[4 marks]

Question 5

By completing the square, solve the equation; $9x^2 + 6x - 17 = 0$

[4 marks]

Question 6

By completing the square, solve the equation; $9x^2 - 6x - 26 = 0$

[4 marks]