Lesson 7

Additional Mathematics
A-Level Pure Mathematics : Year 1
Topics in Algebra

7.1 Sketching Quadratic Curves

Given a quadratic curve, our various algebraic techniques are useful in quickly getting an approximate idea of what its graph looks like, without going to the time and effort of plotting an accurate graph.

7.2 Example

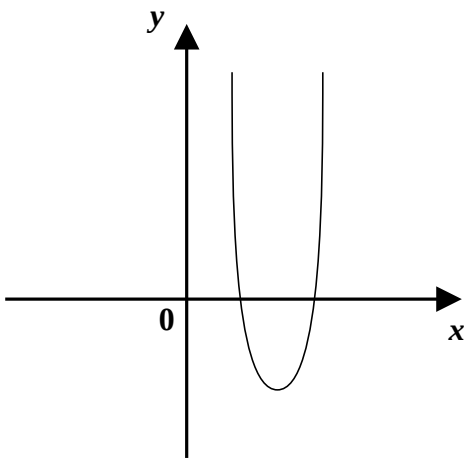
Consider the curve:

$$y = x^2 - 14x + 46$$

(i) Rewrite the equation of the curve in completed square form.

[1 mark]

(ii) Hence add the coordinates of the minimum point to the sketch graph.



[1 mark]

(iii) What is the minimum value of the function $f(x) = x^2 - 14x + 46$?

[1 mark]

(iv) For what value of x does the function $f(x) = x^2 - 14x + 46$ have minimum value ?

[1 mark]

7.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 45

Question 1

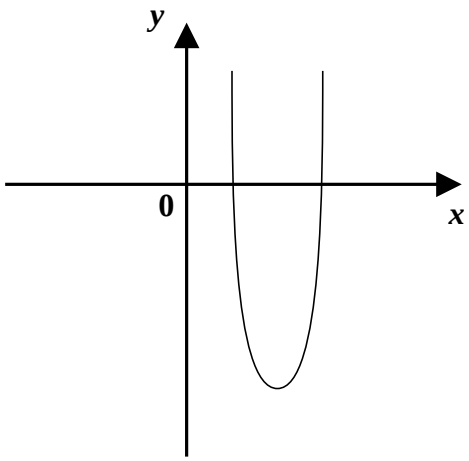
Consider the curve:

$$y = x^2 - 8x + 5$$

(i) Rewrite the equation of the curve in completed square form.

[1 mark]

(ii) Hence add the coordinates of the minimum point to the sketch graph.



[1 mark]

(iii) What is the minimum value of the function $f(x) = x^2 - 8x + 5$?

[1 mark]

Question 2

Given that $f(x) = 3x^2 + 12x + 19$ find the value of the discriminant and explain what it tells you about the graph of $y = f(x)$

[2 marks]

Question 3

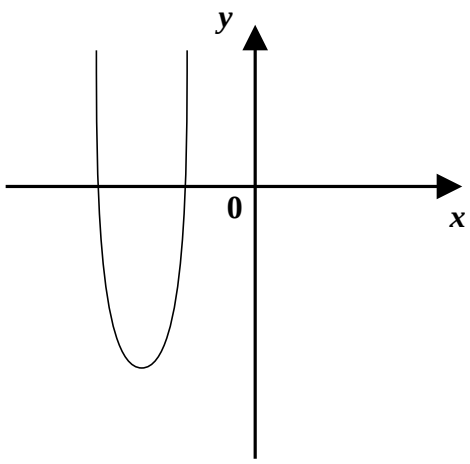
Consider the curve:

$$y = x^2 + 10x + 18$$

(i) Rewrite the equation of the curve in completed square form.

[1 mark]

(ii) Hence add the coordinates of the minimum point to the sketch graph.



[1 mark]

(iii) For what value of x does the function $f(x) = x^2 + 10x + 18$ have its minimum value ?

[1 mark]

Question 4

Given that $f(x) = 3x^2 + 12x + 19$ write $f(x)$ in the form $p(x + q)^2 + r$, where p, q and r are integers to be found.

[3 marks]

Question 5

Without sketching a graph, for each function state its minimum value.

(i) $f(x) = (x - 3)^2 + 11$ **(ii)** $f(z) = \left(z + \frac{1}{4}\right)^2 + \frac{3}{8}$

(iii) $g(x) = 8(x + 7)^2 - 1$ **(iv)** $f(w) = 4\left(w - \frac{5}{2}\right)^2 - \frac{3}{4}$

[4 marks]

Question 6

Without sketching a graph, for each function state the value of x for which the function has minimum value.

(i) $f(x) = (x - 8)^2 + 13$ **(ii)** $f(x) = \left(x - \frac{2}{5}\right)^2 + \frac{4}{7}$

(iii) $g(x) = 7(x + 2.4)^2 - 3.6$ **(iv)** $f(x) = \frac{3}{10}\pi(x - 8)^2 - 32$

[4 marks]

Question 7

Given that $k > 0$, by considering $\left(\sqrt{k} - 1\right)^2$, or otherwise, use algebra to show that $\frac{k + 1}{\sqrt{k}}$ has a least value of 2.

[4 marks]

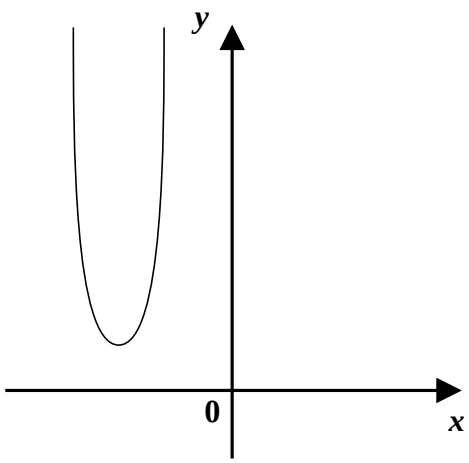
Question 8

Consider the curve, $y = 3x^2 + 30x + 76$

(i) Rewrite the equation of the curve in completed square form.

[3 marks]

(ii) Hence add the coordinates of the minimum point to the sketch graph.



[1 mark]

(iii) What is the minimum value of the function $f(x) = 3x^2 + 30x + 76$?

[1 mark]

Question 9

Show that the quadratic equation $x^2 + (2k + 3)x + k^2 + 3k + 1 = 0$ has two distinct real roots in x, for all values of the constant k.

[4 marks]

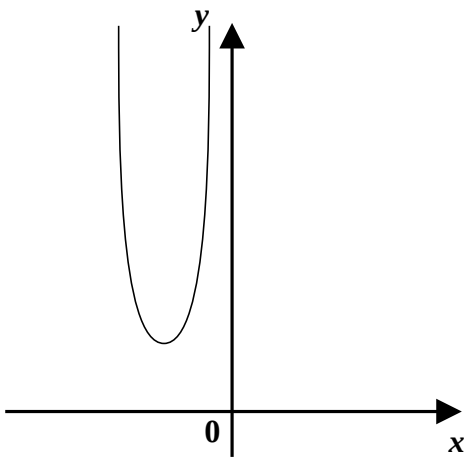
Question 10

Consider the curve, $y = x^2 + x + 1$

(i) Rewrite the equation of the curve in completed square form.
Your answer will have some fractions in it.

[1 mark]

(ii) Hence add the coordinates of the minimum point to the sketch graph.



[1 mark]

(iii) For what value of x does the function $f(x) = x^2 + x + 1$ have its minimum value ?

[1 mark]

Question 11

The cubic curve $y = x^3 - 6x^2 + 12x + B$ can be written in the form

$y = (x - A)^3 - 4$, where A and B are non-zero constants.
Find the value of A and the value of B.

[5 marks]

Question 12
Consider the curve;

$$y = x^2 - 5x - 24$$

In this question we are NOT interested in finding the minimum point.
Instead we wish to know where the graph of this curve crosses the x-axis.

(i) Write the equation of the curve in factorised form.

[1 mark]

(ii) Hence state the coordinates of the two points where the graph of the curve crosses the x-axis.

[2 marks]

(iii) Without further calculation, sketch the curve.

[2 marks]