Lesson 8

Additional Mathematics
A-Level Pure Mathematics : Year 1
Topics in Algebra

8.1 Inequalities

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 45

"Together" Examples
Solve these inequalities:

(i) $5x + 2 \leqslant 9$

(ii) $7 - 3x \geqslant 13$

[1, 2 marks]

(iii) $-2 < 4x + 3 \leqslant 9$

(iv) $\frac{4}{x} < \frac{5}{9}$

[2, 2 marks]

This next question is where the focus falls in the A-Level course.
It's crucial to draw a graph in order to understand properly what is going on.

(v) $x^2 - 2x - 15 < 0$

[3 marks]

8.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 45

Question 1

Solve these inequalities;

(i) $7x - 3 > 25$ **(ii)** $13 - 4x < 11$

[1, 2 marks]

(iii) $3x - 5 > 19 - 5x$ **(iv)** $3(2x + 3) - 4(x - 2) < 11$

[2, 2 marks]

(v) $17 + 7(x - 6) > 45$ **(vi)** $13 < 5x + 3 < 21$

[2, 2 marks]

(vii) $-3 < 5 - 4x \leqslant 25$ **(viii)** $\frac{10}{x} < \frac{5}{12}$

[2, 2 marks]

Question 2

Solve these quadratic inequalities by;

- ☐ First factorising the quadratic into two pairs of brackets.
- ☐ Clearly stating the 'critical values' of the quadratic.
- ☐ Sketching the quadratic.
- ☐ Presenting your final solution with inequality signs clearly displayed.

(i) $x^2 + 9x + 20 \leqslant 0$ **(ii)** $x^2 + 3x - 18 < 0$

[4, 4 marks]

(iii) $x^2 + 13x + 12 > 0$ **(iv)** $x^2 - 11x + 24 \geqslant 0$

[4, 4 marks]

Question 3

The discriminant of the quadratic equation $ax^2 + bx + c = 0$ is $D = b^2 - 4ac$
Each of the following equations is to have two distinct real roots. $\therefore\ D > 0$
By considering this inequality, find the range of k for each of the following,

(i) $x^2 + 6x + k = 0$ **(ii)** $kx^2 + 4x + 2 = 0$

[4, 4 marks]

(iii) $x^2 - 2x + k = 0$ **(iv)** $kx^2 - 6x + 3 = 0$

[4, 4 marks]

Question 4

Consider the following equation;

$$k x^2 - 2x + k = 0$$

This equation has two distinct real roots.

(i) By considering the discriminant show that this implies,

$$(k - 1)(k + 1) < 0$$

[4 marks]

(ii) Hence state the range of values of k.

[4 marks]

Question 5

(i) By completing the square, write the algebraic expression

$$2x^2 - 28x + 87$$

in the form

$$a(x + b)^2 + c$$

where a, b and c are numbers to be found.

[3 marks]

(ii) Hence show that the equation

$$2x^2 - 28x + 87 = 0$$

has solutions of the form

$$x = c \pm d\sqrt{22}$$

where c and d are numbers to be found.

[3 marks]