Lesson 9

Additional Mathematics
A-Level Pure Mathematics : Year 1
Topics in Algebra

9.1 Simultaneous Equations (One linear, one Quadratic)

Solving simultaneous equations where one equation is linear and the other quadratic often come up in examinations because they test several algebraic manipulation skills all in one question:

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$
- Factorising quadratics
- Solving quadratic equations

9.2 Example : The Question

Solve the simultaneous equations,

$$y = 2x - 3 \qquad \text{(line)}$$

$$x^2 + y^2 = 2 \qquad \text{(circle)}$$

9.3 Example : The Solution

We begin by using *the method of substitution.*

$$x^2 + y^2 = 2$$

$$x^2 + (2x - 3)^2 = 2$$

- Expanding brackets, FOIL

$$x^2 + (2x - 3)(2x - 3) = 2$$

$$x^2 + 4x^2 - 6x - 6x + 9 = 2$$

- Gathering together like terms

$$5x^2 - 12x + 9 = 2$$

- Rearranging equations into the form $f(x) = 0$

$$5x^2 - 12x + 7 = 0$$

- Factorising quadratics

$$(5x - 7)(x - 1) = 0$$

- Solving quadratic equations

$$\text{Either } 5x - 7 = 0 \text{ or } x - 1 = 0$$

$$x = 1.4 \qquad x = 1$$

Geometrically, the answers are points where the line intersects the circle.
So complete the solution by using the equation of the line, $y = 2x - 3$, to work out the y values that correspond to x being 1.4, or 1

The points of intersection are $(1.4, -0.2)$ or $(1, -1)$

[5 marks]

9.4 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 30

Question 1
Solve the simultaneous equations

$$y = x - 4$$
$$x^2 + y^2 = 58$$

[5 marks]

Question 2

GCSE Examination Question from June 2007, Paper 3H, Q19 (Edexcel)

Solve the simultaneous equations

$$y = 3x - 1$$

$$x^2 + y^2 = 5$$

[5 marks]

Question 3

A-Level Examination Question from May 2011, C1, Q4 (Edexcel)

Solve the simultaneous equations

$$x + y = 2$$

$$4y^2 - x^2 = 11$$

[5 marks]

Question 4

Solve the simultaneous equations

$$y = x - 7$$

$$x^2 + y^2 = 109$$

[5 marks]

Question 5

A-Level Examination Question from January 2010, C1, Q5 (Edexcel)

Solve the simultaneous equations,

$$y - 3x + 2 = 0$$

$$y^2 - x - 6x^2 = 0$$

[5 marks]

Question 6

Solve the simultaneous equations,

$$y = 2x - 3$$
$$x^2 + y^2 = 18$$

[5 marks]