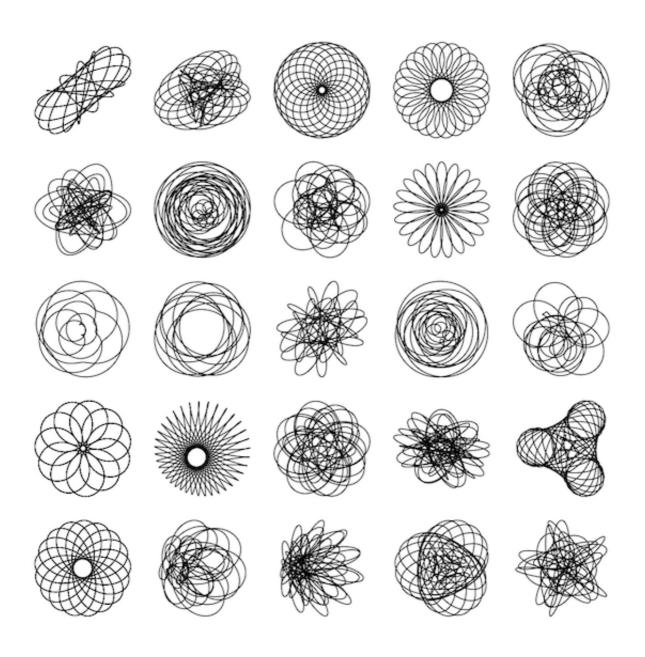
# A-Level Pure Mathematics Year 2

# DIFFERENTIATIO N III



Chain Rule • Product Rule • Quotient Rule

## 1.1 The Chain Rule

The Chain Rule is a rule about differentiation that has similarities with the familiar Differentiation Power Rule for differentiating x raised to the power n.

## The Differentiation Power Rule

If 
$$y = x^n$$
 then  $\frac{dy}{dx} = n x^{n-1}$  where x is a variable and n is a constant.

## 1.2 Example

Find the derivative of  $y = (4x + 5)^2$  by,

- (i) first expanding the brackets, then using "The Differentiation of  $x^n$  Rule"
- (ii) using the "The Chain Rule"

## Answer (i)

$$y = (4x + 5)^{2}$$

$$y = (4x + 5)(4x + 5)$$

$$y = 16x^{2} + 40x + 25$$

$$\frac{dy}{dx} = 32x + 40$$

$$\frac{dy}{dx} = 8(4x + 5)$$

## Answer (ii)

Teaching Video: <a href="http://www.NumberWonder.co.uk/v9028/1.mp4">http://www.NumberWonder.co.uk/v9028/1.mp4</a>



After watching the video write out the chain rule method of solution here

## 1.3 Three For You To Do

Here is the formal statement of The Chain Rule that was given in the video,

The Chain Rule for  $y = [f(x)]^n$ 

If 
$$y = [f(x)]^n$$
 then  $\frac{dy}{dx} = n [f(x)]^{n-1} f'(x)$ 

Now try to apply The Chain Rule to these three further examples. The answers are on the following page so you can immediately check to see if you are correct.

**Try 1** 
$$y = (4x + 5)^{20}$$

[2 marks]

**Try 2** 
$$y = (8x + 1)^5$$

[ 2 marks ]

**Try 3** 
$$y = (4x^3 + 8)^4$$

**Answer to Try 1** 

$$y = (4x + 5)^{20}$$

$$\frac{dy}{dx} = 20(4x + 5)^{19} \times 4$$

$$\frac{dy}{dx} = 80(4x + 5)^{19}$$

Answer to Try 2

$$y = (8x + 1)^{5}$$

$$\frac{dy}{dx} = 5(8x + 1)^{4} \times 8$$

$$\frac{dy}{dx} = 40(8x + 1)^{4}$$

**Answer to Try 3** 

$$y = (4x^{3} + 8)^{4}$$

$$\frac{dy}{dx} = 4(4x^{3} + 8)^{3} \times (12x^{2})$$

$$\frac{dy}{dx} = 48x^{2}(4x^{3} + 8)^{3}$$

## 1.4 Exercise

Marks Available: 35

## **Question 1**

Differentiate  $y = (5x^2 + 7)^3$ 

[2 marks]

**Question 2** 

Differentiate  $y = 5(11 - 6x^2)^5$ 

Differentiate  $y = \sqrt{9 - 5x}$ 

[ 2 marks ]

# **Question 4**

Differentiate the following function,

$$f(x) = 4 (9 + 14x)^{\frac{3}{2}}$$

[2 marks]

# **Question 5**

Differentiate the following function,

$$f(x) = \frac{5}{(7x + 8)^3}$$

Find 
$$\frac{dy}{dx}$$
 when  $y = \frac{5}{3(7-2x)^5}$ 

[ 3 marks ]

# **Question 7**

Differentiate the following function;

$$y = 6 + \frac{1}{(3x^2 + 2)}$$

[ 3 marks ]

# **Question 8**

Consider the curve,

$$y = (2x - 3)^4$$

Find the equation of the tangent to the curve at the point (  $2,\,1$  )

Give your answer in the form y = mx + c

(a) A curve has the following equation,

$$y = (x + 3)^3 - 4(x + 3)$$

Find the coordinates of points on the curve with gradient 8.

[ 3 marks ]

(  $\bf b$  ) Repeat part (a) for the curve with the following equation,

$$y = x^3 - 4x$$

[ 3 marks ]

(c) How are your part (a) and part (b) answers related?

(a) Find the equation of the tangent to the curve,

$$y = \frac{2}{x^2 - 3}$$

at the point  $\left(3, \frac{1}{3}\right)$ 

Give your answer in the form ax + by + c = 0

[4 marks]

(**b**) Find the normal to the curve at the same point. Again, give your answer in the form ax + by + c = 0.

[4 marks]