**A-Level Pure Mathematics: Year 2** 

**Differentiation III** 

#### 2.1 The Product Rule

Given two functions, u(x) and v(x) that are multiplying each other, The Product Rule gives a method of obtaining the derivative of their product. It states that,

$$(u(x) v(x))' = u(x) v'(x) + u'(x) v(x)$$

All of the *x* in brackets are considered to be unnecessary clutter and so the rule is more usually written in the following succinct and elegant form,

**The Product Rule** 

If 
$$f = uv$$
 then  $f' = uv' + u'v$ 

#### 2.2 Example

Differentiate the product of  $x^5$  with  $x^3$  by immediately applying The Product Rule.

Teaching Video: <a href="http://www.NumberWonder.co.uk/v9028/2.mp4">http://www.NumberWonder.co.uk/v9028/2.mp4</a>



Watch the video and then write out the solution here

### 2.3 Exercise

Marks Available: 34

#### **Question 1**

(i) Show how to use The Product Rule to differentiate,

 $y = x^9 \times x^{11}$ 

[ 2 marks ]

(ii) Use algebra to simplify first, then differentiate,

$$y = x^9 \times x^{11}$$

[ 2 marks ]

## **Question 2**

(i) Show how to use The Product Rule to differentiate,

$$y = 3x^5 \times 4x^7$$

[2 marks]

(ii) Use algebra to simplify first, then differentiate,

$$y = 3x^5 \times 4x^7$$

[2 marks]

# **Question 3**

(i) Show how to use The Product Rule to differentiate,

$$y = x \times x$$

[ 2 marks ]

(ii) Use algebra to simplify first, then differentiate,

$$y = x \times x$$

[ 2 marks ]

# **Question 4**

(i) Show how to use The Product Rule to differentiate,

$$y = 8x^{\frac{3}{2}} \times 6x^{\frac{5}{2}}$$

[ 3 marks ]

(ii) Use algebra to simplify first, then differentiate,

$$y = 8x^{\frac{3}{2}} \times 6x^{\frac{5}{2}}$$

# **Question 5**

(i) Show how to use The Product Rule to differentiate,

$$y = x^{-3} \times x^8$$

[2 marks]

(ii) Use algebra to simplify first, then differentiate,

$$y = x^{-3} \times x^8$$

[ 2 marks ]

# **Question 6**

(i) Show how to use The Product Rule to differentiate,

$$y = (x^2 - 1)(x^2 + 1)$$

[ 3 marks ]

(ii) Use algebra to simplify first, then differentiate,

$$y = \left(x^2 - 1\right)\left(x^2 + 1\right)$$

### **Question 7**

Use the product rule to show that;

$$y = x^4 (3x^2 + 1)$$

has a first derivative given by,

$$\frac{dy}{dx} = 2x^3 \left(9x^2 + 2\right)$$

and a second derivative given by,

$$\frac{d^2y}{dx^2} = 6x^2 \left(15x^2 + 2\right)$$

before determining the third derivative.

[6 marks]