**Differentiation III** 

## 3.1 Product In, Product Out (PIPO)

When using The Product Rule, the object before differentiation is a product. It's considered elegant to have an object after the differentiation that's also a product. Furthermore, in tackling optimisation problems in which local minima and maxima are sought, (which correspond to where the derivative, the gradient, is zero) having a product equalling zero (rather than a sum) is a desirable situation. In short, initially applying The Product Rule is often only half of a question; manipulating the algebra to derive an answer in the form of a product is the other.

#### 3.2 Example

Show that the derivative of  $y = x^3 (2x + 5)^3$  can be expressed as,

$$\frac{dy}{dx} = 3x^2(2x+5)^2(4x+5)$$

Teaching Video: <a href="http://www.NumberWonder.co.uk/v9028/3.mp4">http://www.NumberWonder.co.uk/v9028/3.mp4</a>



Watch the video and then write out the solution here

#### 3.3 One For You To Do

Often, in the middle of using The Product Rule, The Chain Rule is required.

The Chain Rule for  $y = [f(x)]^n$ 

If 
$$y = [f(x)]^n$$
 then  $\frac{dy}{dx} = n [f(x)]^{n-1} f'(x)$ 

#### **The Product Rule**

If 
$$f = uv$$
 then  $f' = uv' + u'v$ 

Try this problem, then check your solution with mine on the following page.

**Try 1** Use the product rule to show that the derivative of,

$$y = (2x + 3)^2 (5x - 1)^3$$

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^2(50x + 41)$$

### **Answer to Try 1**

$$y = (2x + 3)^{2} (5x - 1)^{3}$$

$$\frac{dy}{dx} = (2x + 3)^{2} \times 3(5x - 1)^{2} \times 5 + 2(2x + 3)^{1} \times 2 \times (5x - 1)^{3}$$

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^{2} \{15(2x + 3) + 4(5x - 1)\}$$

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^{2} \{30x + 45 + 20x - 4\}$$

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^{2} \{50x + 41\}$$

#### 3.3 Exercise

Marks Available: 35

#### **Question 1**

Use the product rule to show that the derivative of,

$$y = x^5(x-1)^2$$

$$\frac{dy}{dx} = x^4(x-1)(7x-5)$$

# **Question 2**

Use the product rule to show that the derivative of,

$$y = x^7 (6x + 5)^3$$

is,

$$\frac{dy}{dx} = 5x^6(6x + 5)^2(12x + 7)$$

[ **5** marks ]

## **Question 3**

Use the product rule to show that the derivative of,

$$y = (x^2 - 3)(x + 1)^2$$

$$\frac{dy}{dx} = 2(x^2 - 1)(2x + 3)$$

## **Question 4**

Use the product rule to show that the derivative of,

$$y = (4x + 1)^{\frac{3}{2}}(x^2 + 5)$$

is,

$$\frac{dy}{dx} = 2\sqrt{4x+1} \left(7x^2 + x + 15\right)$$

[ 5 marks ]

## **Question 5**

Use the product rule to show that the derivative of,

$$y = (2x - 3)^3 (x^2 + 1)^2$$

$$\frac{dy}{dx} = 2(2x-3)^2(x^2+1)(7x^2-6x+3)$$



Use the product rule to find the derivative of,

$$y = x^3 (5x + 1)^2$$

Write your answer as a product.

[ 5 marks ]

## **Question 7**

Find the *x* component of the coordinates of the stationary points on the curve

$$y = \left(x^2 - 1\right)\sqrt{1 + x}$$

[ 5 marks ]