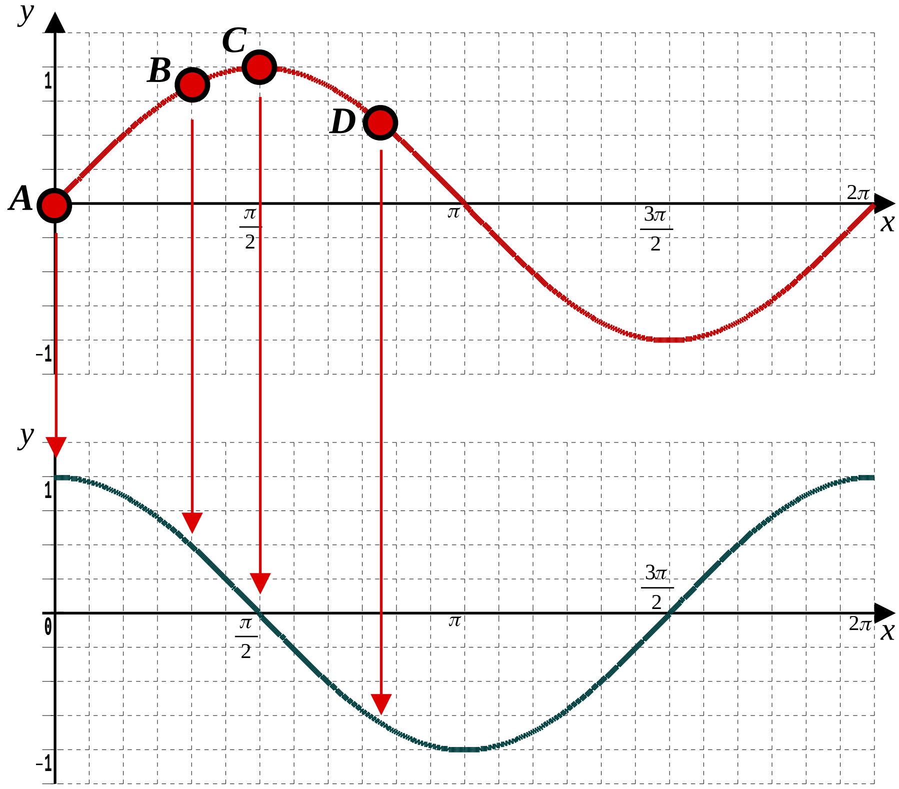
Lesson 8

A-Level Pure Mathematics : Year 2
Differentiation III

8.1 The Sine Function

The upper graph shows the curve $y = \sin x$ with x measured in radians.
Of interest is the gradient of this graph.



Consider what the gradient of the upper graph is doing at the four points labelled.
A : The curve at the origin seems to be sloping like the line $y = x$, gradient 1.
B : The gradient between A and B is positive and reducing in magnitude.
C : At this maximum turning point the gradient has fallen to zero.
D : The gradient between C and D is negative and increasing in magnitude.

Continuing in this manner, and plotting the gradients as a separate graph, the lower graph is obtained. It looks very much like the graph of cosine !

This argument, although not a proof, is an intuitive reasoning of the important result that, provided radians are used,

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Derivative of Sine

If $y = \sin x$ then $\dfrac{dy}{dx} = \cos x$ where x is in radians

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8.2 Differentiating Sine Functions

The Product Rule and The Quotient Rule can be applied to situations where the sine function is involved. So too can The Chain Rule, as follows,

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The Chain Rule for $y = sin(f(x))$

$$\text{If } y = sin(f(x)) \text{ then } \frac{dy}{dx} = cos(f(x)) \times f'(x)$$

8.3 Examples

Differentiate each of the following,

(i) $y = sin^4 3x$ (Chain Rule Example)

(ii) $y = x\,sin\left(x^2\right)$ (Product Rule Example)

(iii) $y = \dfrac{sin\,(2x)}{e^{2x}}$ (Quotient Rule Example)

Teaching Video : http://www.NumberWonder.co.uk/v9028/8.mp4

Watch the video and then write out the solutions here

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[3, 3, 3 marks]

8.4 Exercise

Marks Available : 40

Question 1

Differentiate each of the following with respect to *x*,

(i) $y = 7\, sin\,(5x)$

(ii) $y = 6\, sin\left(3x^3\right)$

[2, 2 marks]

(iii) $y = 2\, sin^3 x$

(iv) $y = \sqrt{sin\, x}$

[2, 2 marks]

(v) $y = 11\, e^{sin\, x}$

(vi) $y = 3\, e^{2\, sin(5x)}$

[2, 2 marks]

(vii) $y = ln\,(sin\, x)$

(viii) $y = sin\,(ln\, x)$

[2, 2 marks]

Question 2

By writing $y = csc\,x$ as $y = (\,sin\,x\,)^{-1}$ and using The Chain Rule, show that,

Derivative of *csc x*

If $y = csc\,x$ then $\dfrac{dy}{dx} = -\,csc\,x\,cot\,x$ where x is in radians

[4 marks]

Question 3

$$f\,(x) = x\,sin\,x$$

(i) Remembering to use radians, find the exact value of $f\left(\dfrac{\pi}{6}\right)$

[2 marks]

(ii) Use The Product Rule to differentiate $f\,(x)$

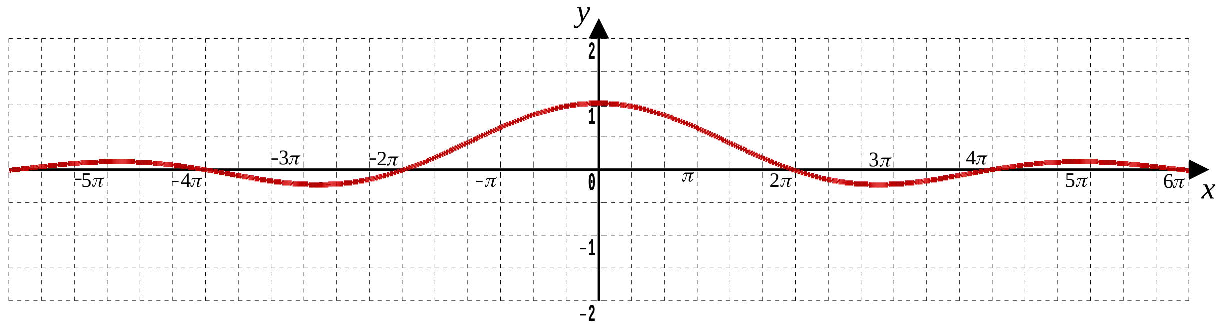
[2 marks]

(iii) Show that $f\,'\left(\dfrac{\pi}{6}\right) = \dfrac{6 + \sqrt{3}\,\pi}{12}$

[2 marks]

Question 4

The graph is of the important function, $f(x) = \dfrac{\sin x}{x}$



(i) The graph suggests a key result about the value of $\underset{x \to 0}{limit} \dfrac{\sin x}{x}$

Explain where one should look on the graph for the result and what it is.

[2 marks]

(ii) Use The Quotient Rule to differentiate $f(x)$

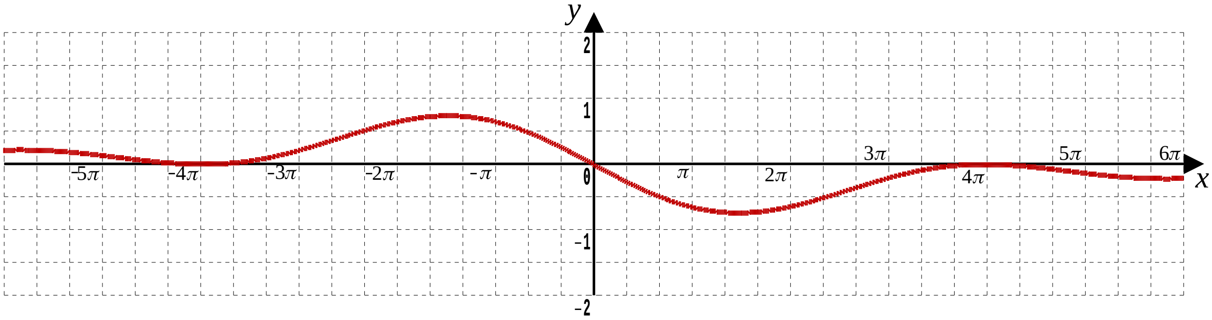
[2 marks]

(iii) Determine the exact value of $f'(\pi)$

[2 marks]

Question 5

The graph is of another important function, $f(x) = \dfrac{\cos x - 1}{x}$



Use the graph to deduce a key result for $\lim_{x \to 0} \dfrac{\cos x - 1}{x}$

Explain where you were looking on the graph to deduce your result.

[2 marks]

Question 6

Prove from first principles that the derivative of *sin x* is *cos x*

Make use of the limit studied in Question 4 and the limit studied in Question 5 along with the following video from "The Math Sorcerer"

Teaching Video : http://www.NumberWonder.co.uk/v9028/8b.mp4

[6 marks]