5.1 Example

The line with equation $y = 3x + 1$ is a tangent to a circle with centre $(30, 21)$

(i) Find the equation of the circle.

[5 marks]

The line with equation $y = 3x + k$, $k \neq 1$, is also a tangent to the circle

(ii) Find the value of the constant k

[2 marks]

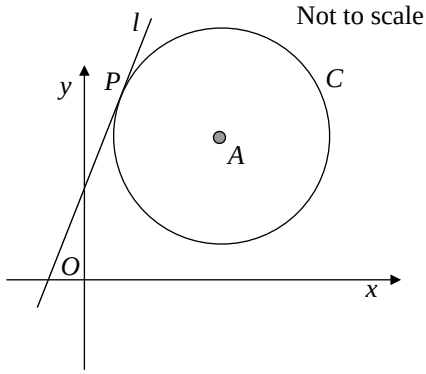
5.2 Revision Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 52

Question 1

A-Level Examination Question, June 2018, Paper 1, Q6 (Edexcel)



The circle C has centre A with coordinates (7, 5)

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P

(a) Show that an equation of the line PA is $2y + x = 17$

[3 marks]

(b) Find an equation for C

[4 marks]

The line with equation $y = 2x + k$, $k \neq 1$, is also a tangent to C

(c) Find the value of the constant k

[3 marks]

Question 2

Given that, $\left(7 - \sqrt{3}\right)\left(5 - \sqrt{3}\right) = a + b\sqrt{3}$, where a and b are integers, find the value of a and the value of b

[2 marks]

Question 3

(i) Complete the square for the following function,

$$f(x) = x^2 - 8x + 23$$

[1 marks]

(ii) Use your part **(i)** answer to explain why the graph of the function will not have any x-axis crossing points

[2 marks]

(iii) Given your previous answers, would you expect the discriminant of the function to be positive, negative or zero ?

[1 mark]

Question 4

Given that, $\dfrac{\sqrt{3}}{3 + 2\sqrt{3}} = m + n\sqrt{3}$, where m and n are integers, find the value of m and the value of n

[3 marks]

Question 5

Given that,

$$5x^2 + 10x - 2 = a(x + b)^2 + c$$

where a, b and c are integers, find the value of a, the value of b and the value of c

[3 marks]

Question 6

Find the set of values of x for which

(i) $3(2x + 1) > 5 - 2x$

[2 marks]

(ii) $2x^2 - 7x + 3 > 0$

[3 marks]

(iii) **both** $3(2x + 1) > 5 - 2x$

and $2x^2 - 7x + 3 > 0$

[1 mark]

Question 7

AS-Level Examination Question, June 2018, Q14 (Edexcel)

The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of C

(ii) the radius of C

[3 marks]

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points

(b) Find the range of values of k

[6 marks]

Question 8

$f(x) = x^2 + (k + 3)x + k$ where k is a real constant

(a) Find the discriminant of $f(x)$ in terms of k

[2 marks]

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k + a)^2 + b$ where a and b are integers to be found.

[2 marks]

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots

[2 marks]

Question 9

Solve the equation, $8^{2x} - 10\left(8^x\right) + 16 = 0$

[3 marks]

Question 10

$$m(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$$

Factorise the quartic polynomial completely.

[6 marks]