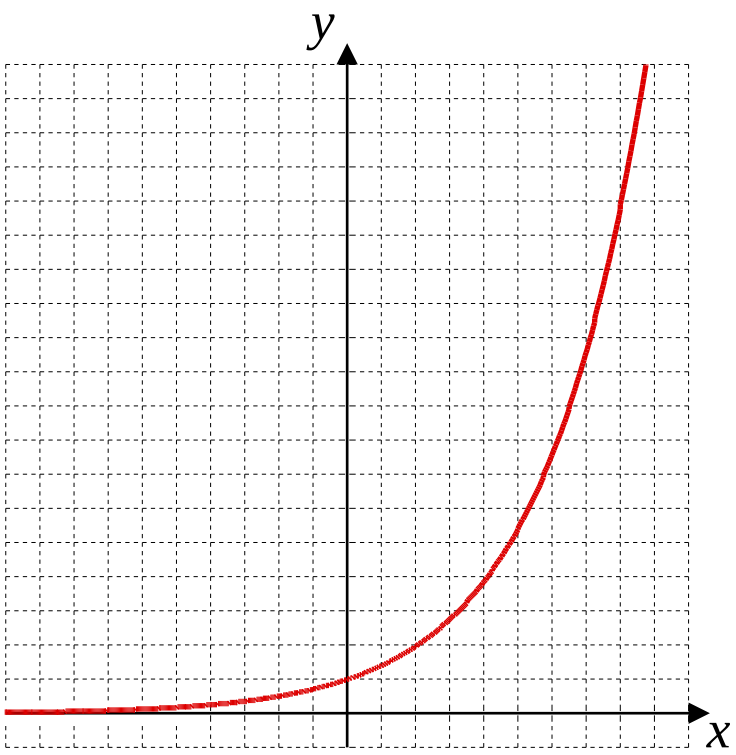
A-Level Mathematics

Year 1

~ Pure Mathematics ~

E X P O N E N T I A L S
a n d
L O G A R I T H M S



The graph of an exponential

E X P O N E N T I A L S
and
L O G A R I T H M S

Lesson 1

A-Level Pure Mathematics : Year 1
Exponentials and Logarithms

1.1 An Exponential Curve and its Inverse

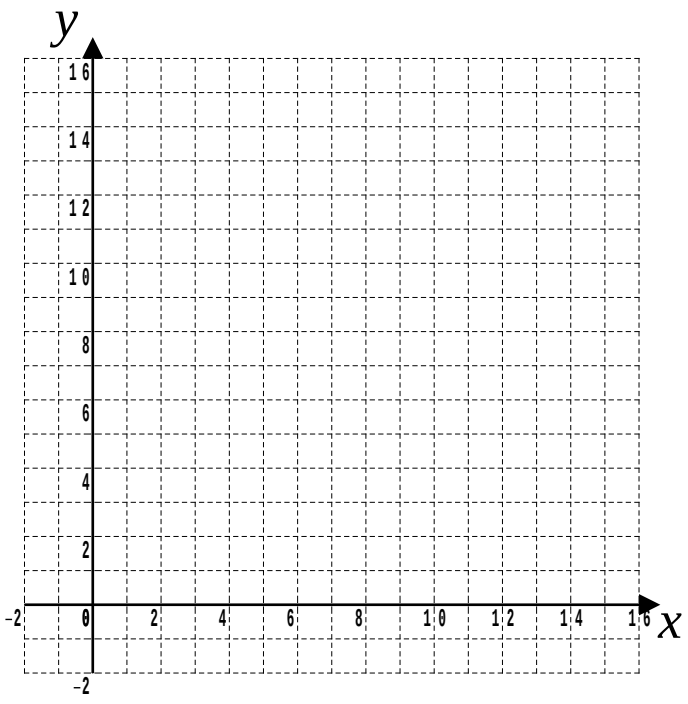
(i) Complete the table of values for the exponential function given by

$$y = 2^x$$

x	-2	-1	0	1	2	3	4
y				2	4		16

[2 marks]

(ii) Graph the function, using the values from the completed table.



[2 marks]

(iii) Use your graph, evidencing your method, to find $2^{2.5}$

[2 marks]

(iv) Use your graph, evidencing your method, to help you solve the equation

$$2^x = 14$$

[2 marks]

(v) Add the line $y = x$ to your graph.

[1 mark]

(vi) Reflect the exponential curve, plotted in part (ii), in the line $y = x$.

[2 marks]

The reflection curve is of the inverse function.
The inverse of an exponential function is a logarithm function.

In this case, the exponential function;

$$y = 2^x$$

has as an inverse, the logarithm function;

$$y = log_2 x$$

(vii) Use your reflection graph, evidencing your method, to find

$$log_2 9$$

[2 marks]

(viii) Use your reflection graph, evidencing your method, to find

$$log_2 16$$

Notice that this means, “2 to what power is 16 ?”

[2 marks]

(ix) This next question is 'off the graph'.
State the value of

$$log_2 64$$

by realising that this means, “2 to what power is 64 ?”

[1 mark]

1.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 40

Question 1

(i) State the value of,

$$log_3 81$$

by realising that this means, "3 to what power is 81 ?"

(ii) State the value of,

$$log_2 32$$

(iii) State the value of,

$$log_{10} 1000000$$

(iv) State the value of,

$$log_4 64$$

[4 marks]

Question 2

(i) State the value of,

$$log_3\left(\frac{1}{9}\right)$$

by realising that this means "3 to what power is $\frac{1}{9}$?"

(ii) State the value of,

$$log_2\left(\frac{1}{8}\right)$$

(iii) State the value of,

$$log_2 0.25$$

(iv) State the value of,

$$log_3\left(\frac{1}{27}\right)$$

[4 marks]

Question 3

(i) Find x given that,

$$log_x 125 = 3$$

by realising that this means, “what to the power 3 is 125 ?”

(ii) Find x given that,

$$log_x 81 = 4$$

(iii) Find x given that,

$$log_x 64 = 2$$

(iv) Find x given that,

$$log_x 9 = 0.5$$

[4 marks]

Question 4

(i) Find x given that,

$$log_2 x = 3$$

by realising that this means, “2 to the power 3 is what ?”

(ii) Find x given that,

$$log_{10} x = 5$$

(iii) Find x given that,

$$log_2 x = -3$$

(iv) Find x given that,

$$log_9 x = \frac{3}{2}$$

[4 marks]

Question 5

(i) Find x given that,

$$log_{10}\,10000 \;=\; x$$

by realising that this means, “10 to what power is 10000 ?”

(ii) Find x given that,

$$log_{10}\,100\,000\,000 \;=\; x$$

(iii) Find x given that,

$$log_{10}\,10 \;=\; x$$

(iv) Find x given that,

$$log_{10}\left(\frac{1}{1000}\right) \;=\; x$$

(v) Find x given that,

$$log_{10}\;0.01 \;=\; x$$

(vi) Find x given that,

$$log_{10}\;\sqrt{10} \;=\; x$$

(vii) Find x given that,

$$log_{10}\;1 \;=\; x$$

(viii) Find x given that,

$$log_{8}\;2 \;=\; x$$

(ix) Find x given that,

$$log_{5}\left(\frac{1}{125}\right) \;=\; x$$

[9 marks]

Question 6

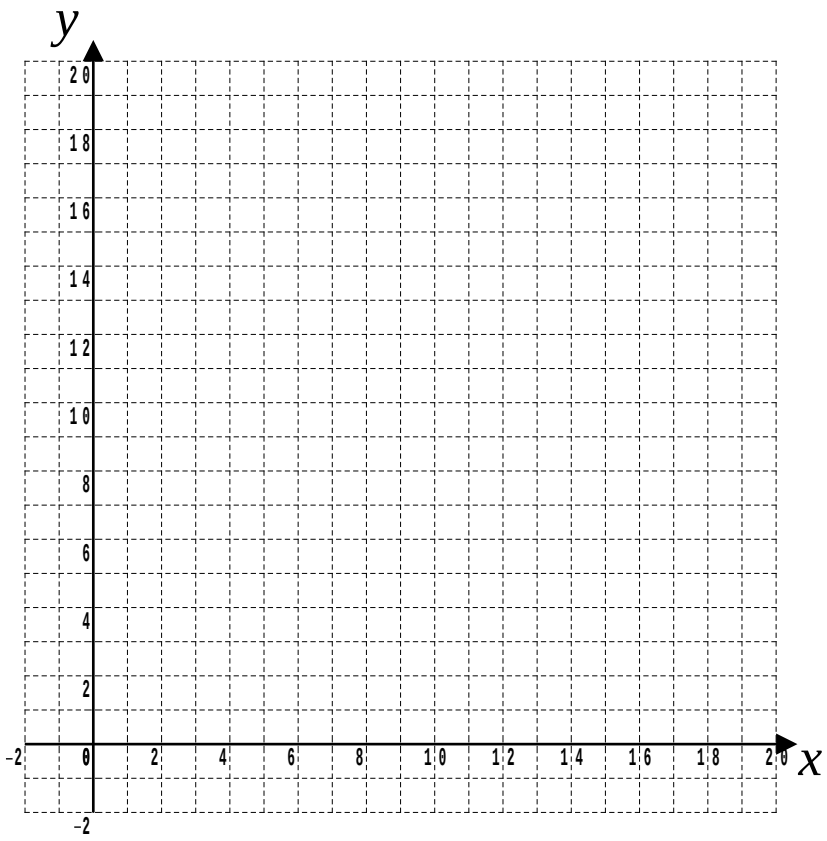
(i) Complete the table of values for the exponential function given by

$$y = 1.2^x$$

x	-2	-1	0	1	2	3	4	5	6	7	8	12	16
y				1.2	1.4		2.1						

[3 marks]

(ii) Graph the function, using the values from the completed table.



[2 marks]

(iii) Use your graph, evidencing your method, to find

$$1.2^{6.5}$$

[2 marks]

(iv) Use your graph, evidencing your method, to help you solve the equation

$$1.2^x = 12$$

[2 marks]

(v) Add the line $y = x$ to your graph.

[1 mark]

(vi) Reflect the exponential curve, plotted in part (ii), in the line $y = x$.

[1 mark]

This reflected curve is the inverse function.
The inverse of an exponential function is a logarithm function.

In this case, the exponential function;

$$y = 1.2^x$$

has as an inverse, the logarithm function;

$$y = log_{1.2}\, x$$

(vii) Use your reflection graph, evidencing your method, to find

$$log_{1.2}\, 8$$

[2 marks]

(viii) Use your reflection graph, evidencing your method, to find

$$log_{1.2}\, 18$$

[2 marks]