Lesson 10

A-Level Pure Mathematics : Year 1
Exponentials and Logarithms

10.1 Miscellaneous Old Examination Questions

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available: 42

Question 1

Solve the equation, $log_5 (3t + 8) = 2.1$

Round your answer to 3 significant figures.

[3 marks]

Question 2

Consider the formula

$$r = 0.1\,e^{0.5t + 1}$$

By first multiplying through by 10, rearrange the formula to make *t* the subject.

[3 marks]

Question 3

The temperature, T °C, of a cup of tea is given by

$$T = 55e^{-0.125t} + 20 \qquad t \geqslant 0$$

where t is the time in minutes since measurements began

(a) Briefly explain why $t \geqslant 0$

[1 mark]

(b) State the starting temperature of the cup of tea.

[1 mark]

(c) Find the time at which the temperature of the tea is 50 °C, giving your answer in minutes and seconds to the nearest second.

[3 marks]

(d) By sketching a graph or otherwise, explain why the temperature of the tea will never fall below 20 °C

[2 marks]

Question 4

The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

[2 marks]

A second dose of 15 mg is given 5 hours after the first dose has been given.
Using the same model for the second dose,

(b) show that the total amount of the antibiotic in the bloodstream 2 hours after the second dose is given is between 13.5 mg and 14 mg.
Give your answer to 3 decimal places.

[2 marks]

Question 5

The equation of a curve is

$$y = 20x^n$$

where n is a constant.

The point $(15, 590)$ is on the curve.
Calculate the value of n

[5 marks]

Question 6

The population, P, of a colony of endangered Caledonian owlet-nightjars can be modelled by the equation $P = a\,b^t$ where a and b are constants and t is the time, in months, since the population was first recorded.

A graph of $\log_{10} P$ against t is linear and has a line of best fit that passes through the point (0, 2) on the y-axis and also the point, twenty months later, (20, 2.2)

(a) Determine the equation of the straight line through the two points.

[3 marks]

(b) Work out the value of a and interpret this value in the context of the model

[3 marks]

(c) Work out the value of b, giving your answer correct to 3 decimal places.

[2 marks]

(d) Find the population predicted by the model when $t = 30$

[1 mark]

Question 7

The following table gives estimates of the world population in different years.

Date	1400	1500	1600	1700	1800
Population (millions)	375	460	540	640	945

Sam is trying to model the population by an equation of the form

$$y = A e^{kt}$$

where y is the population in millions and t is the number of centuries after 1400.

Sam decides to find values of A and k that gives the exact values for 1400 and 1800

(a) Show that Sam's value for A is 375

[1 mark]

(b) Find Sam's value for k

[4 marks]

(c) Does Sam's formula give reasonable value for the population in 1600 ?
Justify your answer.

[2 marks]

Question 8

Solve the equation, $2^{2x} + 2^3 = 2^{x+3}$

[4 marks]