Lesson 11

A-Level Pure Mathematics : Year 1
Exponentials and Logarithms

Revision

11.1 Summary

The First Rule

$$\log_c(ab) = \log_c a + \log_c b$$

The Second Rule

$$\log_c\left(\frac{a}{b}\right) = \log_c a - \log_c b$$

The Third Rule

$$\log_c a^n = n \log_c a$$

The "Jump Out of Logs" Manoeuvre

$$\log_c a = b$$
$$\Leftrightarrow \quad c^b = a$$

Two Special Results

$$\log_c c = 1 \qquad \log_c 1 = 0$$

11.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 40

Question 1

Solve the equation

$$7^x = 5$$

Give your answer correct to 3 decimal places.

[2 marks]

Question 2

By first using a Law of Indices, or otherwise, solve the equation,

$$5^x \times 5^{2x+1} = 10$$

Give your answer to three decimal places.

[3 marks]

Question 3

Solve the following equation

$$log_5(2x + 3) - log_5 x = log_5 8$$

[3 marks]

Question 4

Solve the following equation

$$2\,log_7(x - 6) - log_7 x = log_7 3$$

[5 marks]

Question 5

Solve the equation

$$2^{2x+1} - 2^{x} = 15$$

giving your solutions correct to three decimal places.

[5 marks]

Question 6

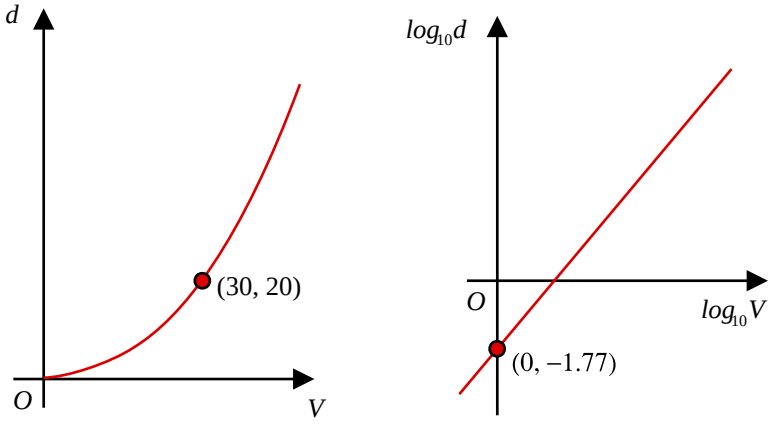
A-Level Examination Question from June 2019, Paper 2, Q9 (Edexcel)

A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of V km h^{-1}

Graphs of d against V and $log_{10} d$ against $log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.



(a) Explain how the rightmost graph would lead the engineer to believe that the braking distance should be modelled by the formula,

$d = k V^n$ where k and n are constants with $k \approx 0.017$

[3 marks]

Using the information given in the leftmost graph, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

[3 marks]

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead.
It takes him 0.8 seconds to react before applying the brakes.

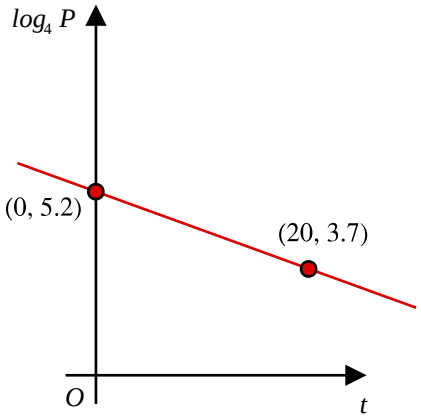
(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

[3 marks]

Question 7

The number of people, P, left in a concert venue t minutes after the band stops playing can be modelled by an equation of the form $P = a\,b^t$

The diagram shows the graph of $log_4 P$ against t



(a) Write down the equation of the line shown on the graph.

[2 marks]

(b) Find the number of people in the venue when $t = 0$, correct to 3 significant figures.

[1 mark]

(c) Find the values of a and b in the model, correct to 3 significant figures.

[2 marks]

(d) Find the number of people half an hour after the band stopped playing.

[1 mark]

Question 8

A-Level Examination Question from May 2016, Paper C2, Q8 (Edexcel)

(i) Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, \qquad a > 2$$

express b in terms of a

[3 marks]

(ii) Solve the equation

$$2^{2x+5} - 7\left(2^{x}\right) = 0$$

giving your answer to 2 decimal places.

[4 marks]