Lesson 13

A-Level Pure Mathematics : Year 1
Exponentials and Logarithms

Later Date Revision

13.1 The Rules Of Logs

The First Rule

$$\log_c(ab) = \log_c a + \log_c b$$

The Second Rule

$$\log_c\left(\frac{a}{b}\right) = \log_c a - \log_c b$$

The Third Rule

$$\log_c a^n = n \log_c a$$

The "Jump Out of *logs*" Manoeuvre

$$\log_c a = b \quad \Leftrightarrow \quad c^b = a$$

Two Special Results

$$\log_c c = 1 \qquad \log_c 1 = 0$$

13.2 STARTER

- Toby wishes to solve the equation

$$3^x = -1$$

By sketching a graph, explain why this equation has no solutions.

- Karen wishes to solve the equation

$$x = log(-2.4)$$

By sketching a graph, explain why this equation has no solutions.

13.3 The Worked Examples

Solve for x,

(i) $9^x - 2 \times 3^{1+x} - 7 = 0$

(ii) $2\log_2 x - 3 = \log_2(3x - 10)$

(iii) $3^{2x-1} = 2^{x+4}$

(iv) $4^{2x-1} = 2^{x+4}$

Answer (i)

$$9^x - 2 \times 3^{1+x} - 7 = 0$$
$$\left(3^2\right)^x - 2 \times 3 \times \left(3^x\right) - 7 = 0$$
$$\left(3^x\right)^2 - 6\left(3^x\right) - 7 = 0$$
$$z^2 - 6z - 7 = 0$$
$$(z - 7)(z + 1) = 0$$

Either $z - 7 = 0$ or $z + 1 = 0$

$$z = 7 \qquad z = -1$$
$$3^x = 7 \qquad 3^x = -1$$
$$\log 3^x = \log 7 \qquad \text{no solutions}$$
$$x \log 3 = \log 7$$
$$x = \frac{\log 7}{\log 3}$$
$$x = 1.771$$

Answer (ii)

$$2\log_2 x - 3 = \log_2(3x - 10)$$
$$2\log_2 x - \log_2(3x - 10) = 3$$
$$\log_2 x^2 - \log_2(3x - 10) = 3$$
$$\log_2\left(\frac{x^2}{3x - 10}\right) = 3$$
$$2^3 = \frac{x^2}{3x - 10}$$
$$24x - 80 = x^2$$
$$x^2 - 24x + 80 = 0$$
$$(x - 20)(x - 4) = 0$$

Either $x = 20$ or $x = 4$

Answer (iii)

$$3^{2x-1} = 2^{x+4}$$
$$\log 3^{2x-1} = \log 2^{x+4}$$
$$(2x - 1)\log 3 = (x + 4)\log 2$$
$$2x\log 3 - \log 3 = x\log 2 + 4\log 2$$
$$2x\log 3 - x\log 2 = 4\log 2 + \log 3$$
$$x(2\log 3 - \log 2) = \log 2^4 + \log 3$$
$$x\left(\log 3^2 - \log 2\right) = \log(16 \times 3)$$
$$x\log\left(\frac{9}{2}\right) = \log(48)$$
$$x = \frac{\log 48}{\log\left(\frac{9}{2}\right)}$$

Answer (iv)

$$4^{2x-1} = 2^{x+4}$$
$$\left(2^2\right)^{2x-1} = 2^{x+4}$$
$$2^{4x-2} = 2^{x+4}$$
$$4x - 2 = x + 4$$
$$3x = 6$$
$$x = 2$$

13.4 Exercise

Question 1

Solve the equation

$$3^x (3^x + 4) = 5$$

[4 marks]

Question 2

Solve the equation

$$3^x = 65$$

Give your answer correct to 3 significant figures.

[3 marks]

Question 3

By first using a Law of Indices, or otherwise, solve the equation,

$$7^{3x} \times 7^{2x+3} = 43$$

Give your answer to three decimal places.

[4 marks]

Question 4

Solve the following equation, giving the exact answer.

$$log_2(5x + 3) - log_2 x = log_2 12$$

[3 marks]

Question 5

Solve the following equation

$$2\log_7(x-4) - \log_7 x = \log_7 2$$

[5 marks]

Question 6

Solve the equation,

$$5^{2x} - 13(5^x) + 22 = 0$$

[5 marks]

Question 7

Solve the equation

$$log_2(3x + 1) + log_2(2x) = 3$$

[6 marks]

Question 8

(i) Given that

$$9^x = 3^a$$

express a in terms of x

(ii) Hence, or otherwise, solve the following equation

$$9^x - 2 \times 3^{x+1} + 8 = 0$$