Lesson 3

A-Level Pure Mathematics : Year 1
Exponentials and Logarithms

3.1 Consolidation Exercise (Non Calculator)

Any solution based entirely on graphical or numerical methods is not acceptable
Marks Available : 70

Question 1
Solve these equations;

(i) $5^x = 125$ **(ii)** $2^x = \frac{1}{8}$

(iii) $9^x = 3$ **(iv)** $17^x = 1$

(v) $\left(\sqrt{2}\right)^x = 8$ **(vi)** $27^x = \frac{1}{9}$

[6 marks]

Question 2
Determine the value of;

(i) 5^{-2} **(ii)** $8^{\frac{2}{3}}$

(iii) 13^0 **(iv)** $9^{-\frac{3}{2}}$

(v) $\left(\sqrt{6}\right)^4$ **(vi)** $100000000^{0.25}$

[6 marks]

Question 3

Solve these equations;

(i) $log_4 64 = x$

(ii) $log_3 \left(\frac{1}{81}\right) = x$

(iii) $log_9 27 = x$

(iv) $log_7 7 = x$

(v) $log_8 1 = x$

(vi) $log_{100} 10 = x$

[6 marks]

Question 4

Solve these equations;

(i) $log_x 8 = 3$

(ii) $log_x 7 = 0.5$

(iii) $log_x \left(\frac{1}{9}\right) = -1$

(iv) $log_x 1000 = 3$

(v) $log_x 25 = \frac{2}{3}$

(vi) $log_x 81 = 4$

[6 marks]

Question 5

Solve these equations;

(i) $log_2 x = 5$ **(ii)** $log_5 x = -1$

(iii) $log_8 x = \frac{5}{3}$ **(iv)** $log_3 x = -3$

(v) $log_{0.5} x = 4$ **(vi)** $log_{\sqrt{10}} x = -4$

[6 marks]

Question 6

(i) Use the rules of logs to prove that,

$$3\,log_x 8 - 2\,log_x 4 = log_x 32$$

[3 marks]

(ii) Hence, or otherwise, determine the integer value of,

$$3\,log_2 8 - 2\,log_2 4$$

[2 marks]

Question 7

Use the rules of logs to prove that,

$$2\log_7 2 + 3\log_7 5 = \log_7 500$$

[4 marks]

Question 8

Given that a and b are positive constants, solve the simultaneous equations

$$a = 8b$$

$$\log_2 a + 2\log_2 b = 4$$

Give your answers as exact numbers.

[6 marks]

Question 9

AS-Level Examination Question from May 2018, Paper 1, Q5 (Edexcel)

A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is as follows;

$$2\log_2 x - \log_2 \sqrt{x} = 3$$

$$2\log_2\left(\frac{x}{\sqrt{x}}\right) = 3 \qquad \text{using the subtraction law for logs}$$

$$2\log_2\left(\sqrt{x}\right) = 3 \qquad \text{simplifying}$$

$$\log_2 x = 3 \qquad \text{using the power law for logs}$$

$$x = 3^2 \qquad \text{using the definition of a log}$$

$$x = 9$$

(a) Identify two errors made by this student, giving a brief explanation of each.

[2 marks]

(b) Write out the correct solution.

[3 marks]

Question 10

(i) Write down the value of $\log_{36} 6$

[1 mark]

(ii) Express $3\log_a 2 + \log_a 13$ as a single logarithm to base a.

[3 marks]

Question 11

A-Level Examination Question from 2018, Specimen Paper 1, Q5 (Edexcel)

$$f(x) = x^3 + ax^2 - ax + 48, \text{ where } a \text{ is a constant}$$

Given that $f(-6) = 0$,

(a) **(i)** show that $a = 4$

[2 marks]

(ii) express $f(x)$ as a product of two algebraic factors.

[2 marks]

Given that $2\log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3$

(b) show that $x^3 + 4x^2 - 4x + 48 = 0$

[4 marks]

(c) hence explain why

$$2\log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3$$

has no real roots.

[2 marks]

Question 12

(i) Use the rules of logs to prove that,

$$5\log_3 x - 2\log_3 3x + \log_3 72 = 3\log_3 2x$$

[4 marks]

(ii) Hence, or otherwise, prove that,

$$5\log_2 x - 2\log_2 3x + \log_2 72 = 3\,(1 + \log_2 x)$$

[2 marks]