Lesson 4

A-Level Pure Mathematics : Year 1
Exponentials and Logarithms

4.1 Taking the *log* of both sides

Given an equation, the "golden rule" of algebra is to "do the same to both sides".
Taking the *log* (of any base) of both sides is sometimes the key to solving an equation that has x as an index.

4.2 A Worked Example

Find the solution to the equation $2^{x+1} = 5^x$
Give your answer accurate to three decimal places.

[4 marks]

Solution :

$$2^{x+1} = 5^x$$

Take the natural logarithm of both sides

$$\ln 2^{x+1} = \ln 5^x$$

$$(x + 1)\ln 2 = x \ln 5$$

$$x \ln 2 + \ln 2 = x \ln 5$$

$$x \ln 5 - x \ln 2 = \ln 2$$

$$x(\ln 5 - \ln 2) = \ln 2$$

$$x = \frac{\ln 2}{\ln 5 - \ln 2}$$

$$x = 0.756 \quad \text{to three decimal places}$$

[4 marks]

Note : It would be fine to take either $\log_2$ or $\log_5$ of both sides.
Indeed, that would save a little working.
The standard approach is to use *ln* as there is a handy special calculator button that avoids having to type in a base.

Note : $\ln = \log_e$ where $e = 2.718...$ which is an irrational number like π or $\sqrt{2}$

Note : On some calculators there may be a button marked simply *log*
This button will actually be $\log_{10}$

Note : In some textbooks, including older A-Level texts, $\log_{10}$ may be written *lg*

4.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 40

Question 1

Solve this equation, giving your answer to three decimal places,

$$2^{x+5} = 3^{x}$$

[4 marks]

Question 2

By first using a Law of Indices, or otherwise, solve the equation,

$$3^{4x} \times 3^{5-x} = 2^{x}$$

Give your answer to three significant figures.

[4 marks]

Question 3

Find the solution to the equation,

$$\frac{1}{3^{2x-1}} = 4^x$$

Give your answer accurate to three decimal places.

[4 marks]

Question 4

Solve, correct to 3 decimal places, the equation

$$3^x - 5 = 0$$

[3 marks]

Question 5

(i) Sketch the curve

$$y = 2^x$$

[2 marks]

(ii) Hence or otherwise, discuss trying to solve the equation

$$2^x = -0.5$$

[2 marks]

Question 6

Use the fact that, $ln(5 \times 3^x) = ln\,5 + ln\,3^x$ to assist in solving the equation,

$$5 \times 3^x = 4^{3x}$$

Give your answer accurate to three decimal places.

[4 marks]

Question 7

Solve the equation,

$$3 \times 5^{x+2} = 8^{x}$$

Give your answer accurate to three decimal places.

[4 marks]

Question 8

(i) Solve,

$$log_2 x = 6$$

[1 mark]

(ii) Solve,

$$log_5 x = 0.5$$

Give your answer to three decimal places.

[1 mark]

(iii) Solve,

$$log_2 x = -4$$

Give the exact answer.

[1 mark]

Question 9

Solve this equation, giving your answer to three decimal places,

$$3^{2x+5} = 4^{x-2}$$

[5 marks]

Question 10

Solve the equation,

$$4 \times 3^{2x+1} = 5^{x}$$

Give your answer accurate to three decimal places.

[5 marks]