Lesson 5

A-Level Pure Mathematics : Year 1
Exponentials and Logarithms

5.1 Index Equations

Example 1

Solve the equation, $3^x = 81$

[1 mark]

The solution is obviously 4 but the real question is "How could the answer be obtained in a systematic manner ?"

$$3^x = 81$$
$$\ln 3^x = \ln 81$$
$$x \ln 3 = \ln 81$$
$$x = \frac{\ln 81}{\ln 3}$$
$$x = 4$$

[1 mark]

Example 2

Solve the equation, $9^x - 3^{x+1} + 2 = 0$

After some thought you may spot one value of x that makes this true.
However, there is a second solution, much more elusive.
It's an irrational number and so needs a methodical method to track down.

$$9^x - 3^{x+1} + 2 = 0$$
$$\left(3^2\right)^x - 3^{x+1} + 2 = 0$$
$$3^{2x} - 3^x \times 3^1 + 2 = 0$$
$$\left(3^x\right)^2 - 3\left(3^x\right) + 2 = 0$$
$$z^2 - 3z + 2 = 0 \quad \text{by letting } z = 3^x$$
$$(z - 2)(z - 1) = 0$$
$$\therefore \text{ Either } z = 2 \quad \text{or} \quad z = 1$$
$$3^x = 2 \quad \text{or} \quad 3^x = 1$$
$$\ln 3^x = \ln 2 \quad \text{or} \quad x = 0$$
$$x = \frac{\ln 2}{\ln 3}$$
$$x = 0.631$$

[5 marks]

5.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Question 1

Solve the equation

$$2^x = 3$$

Give your answer accurate to 3 decimal places.

[2 marks]

Question 2

A-Level Examination Question from June 2008, paper C2, Q4 (Edexcel)

(a) Find, to 3 significant figures, the value of x for which

$$5^x = 7$$

[2 marks]

(b) Solve the equation,

$$5^{2x} - 12\left(5^x\right) + 35 = 0$$

[3 marks]

Question 3

Show that the equation

$$25^x - 5^{x+1} + 4 = 0$$

can be written in the form

$$\left(5^x - 4\right)\left(5^x - 1\right) = 0$$

and hence solve the equation.

[5 marks]

Question 4

Without using logarithms, solve the equation, $4^{2x+2} = 8^{x-5}$

[3 marks]

Question 5

Without using logarithms, solve the equation, $9^x = 27^{2-x}$

[3 marks]

Question 6

Solve the equation, $9^x - 3^{x+2} + 20 = 0$

Give your answers correct to 3 decimal places.

[5 marks]

Question 7

Solve the equation, $3^x \times 3^{2x+1} = 9^x$

[3 marks]

Question 8

Solve the equation, $16^x \times 8^{4x+3} = 4^x$

Give your answer as an exact fraction.

[3 marks]

Question 9

A-Level Examination Question from January 2007, paper C2, Q4 (Edexcel)

Solve the equation, $5^x = 17$

Give your answer to 3 significant figures.

[2 marks]

Question 10

Solve for x

$$9^x - 2 \times 3^{x+1} + 8 = 0$$

[5 marks]

Question 11

Solve for x

$$4^x - 6\left(2^x\right) + 5 = 0$$

[5 marks]

Question 12

A-Level Examination Question from May 2015, paper C1, Q7 (Edexcel)

Given that, $y = 2^x$

(a) express 4^x in terms of y

[1 mark]

(b) Hence, or otherwise, solve

$$8\left(4^x\right) - 9\left(2^x\right) + 1 = 0$$

[4 marks]

Question 13

Show that the equation

$$8^x - 2^{x+6} = 0$$

can be written in the form

$$2^x (2^{2x} - 64) = 0$$

and hence solve the equation.

[4 marks]