### 6.1 Extraneous Solutions

When solving equations involving the logarithm function, it's important to verify that the answers obtained after a sequence of algebraic manipulations do, in fact, satisfy the original equation.
This stems from the fact that certain manipulations can turn a false statement into a true statement. As an example, consider squaring,

$$3 = -3 \qquad \text{(False)}$$
$$3^2 = (-3)^2 \qquad \text{Square both sides}$$
$$9 = 9 \qquad \text{(True)}$$

Here is an illustration of what can happen when a variable is involved,

$$x + 3 = -5 \qquad \left( \text{This has the solution set } \{-8\} \right)$$
$$(x + 3)^2 = 25 \qquad \left( \text{This has the solution set } \{-8,\ 2\} \right)$$

Squaring both sides caused an ***extraneous solution*** to be added.

### 6.2 Example

Solve for $x$, carefully checking for, and discarding, any extraneous solutions,

$$log_6(x + 5) + log_6(x - 4) = 2$$

**[ 4 marks ]**

## 6.3 Exercise

*Any solution based entirely on graphical or numerical methods is not acceptable*

Marks Available : 88

**Question 1**

*A-Level Examination Question from January 2007, paper C2, Q4 (Edexcel)*

Solve the equation

$$5^x = 17$$

giving your answer to 3 significant figures.

**[ 3 marks ]**

**Question 2**

*A-Level Examination Question from June 2005, paper C2, Q2 (Edexcel)*

**( a )** Solve

$$5^x = 8$$

giving your answer to 3 significant figures.

**[ 3 marks ]**

**( b )** Solve

$$\log_2(x + 1) - \log_2 x = \log_2 7$$

**[ 3 marks ]**

**Question 3**

*A-Level Examination Question from May 2011, paper C2, Q3 (Edexcel)*

Find, giving your answers to 3 significant figures where appropriate, the value of $x$ for which;

**( a )** $5^x = 10$

**[ 2 marks ]**

**( b )** $log_3(x - 2) = -1$

**[ 2 marks ]**

**Question 4**

*A-Level Examination Question from January 2005, paper C2, Q3 (Edexcel)*

Find, giving your answers to 3 significant figures where appropriate, the value of $x$ for which;

**( a )**

$$3^x = 5$$

**[ 3 marks ]**

**( b )**

$$log_2(2x + 1) - log_2 x = 2$$

**[ 2 marks ]**

**Question 5**

*A-Level Examination Question from May 2007, paper C2, Q6 (Edexcel)*

**( a )** Find, to 3 significant figures, the value of $x$ for which,

$$8^x = 0.8$$

**[ 2 marks ]**

**( b )** Solve the equation

$$2\log_3 x - \log_3 7x = 1$$

**[ 4 marks ]**

**Question 6**

*A-Level Examination Question from June 2003, paper P2, Q1 (Edexcel)*

**( a )** Simplify, $\dfrac{x^2 + 4x + 3}{x^2 + x}$

**[ 2 marks ]**

**( b )** Find the value of $x$ for which

$$\log_2\left(x^2 + 4x + 3\right) - \log_2\left(x^2 + x\right) = 4$$

**[ 4 marks ]**

**Question 7**

*A-Level Examination Question from January 2010, paper C2, Q5 (Edexcel)*

**( a )** Find the positive value of $x$ such that,

$$\log_x 64 = 2$$

**[ 2 marks ]**

**( b )** Solve for $x$

$$\log_2(11 - 6x) = 2\log_2(x - 1) + 3$$

**[ 6 marks ]**

**Question 8**

*A-Level Examination Question from January 2009, paper C2, Q4 (Edexcel)*

Given that $0 < x < 4$ and

$$\log_5(4 - x) - 2\log_5 x = 1$$

find the value of $x$.

**[ 6 marks ]**

**Question 9**

*A-Level Examination Question from June 2008, paper C2, Q4 (Edexcel)*

**( a )** Find, to 3 significant figures, the value of $x$ for which

$$5^x = 7$$

**[ 2 marks ]**

**( b )** Solve the equation

$$5^{2x} - 12\left(5^x\right) + 35 = 0$$

**[ 4 marks ]**

**Question 10**

*A-Level Examination Question from January 2012, paper C2, Q4 (Edexcel)*

Given that $y = 3x^2$

**( a )** show that $\log_3 y = 1 + 2\log_3 x$

**[ 3 marks ]**

**( b )** Hence, or otherwise, solve the equation

$$1 + 2\log_3 x = \log_3(28x - 9)$$

**[ 3 marks ]**

**Question 11**

*A-Level Examination Question from January 2011, paper C2, Q8 (Edexcel)*

Sketch the graph of $y = 7^x$ showing the coordinates of any points at which the graph crosses the axes.

**[ 2 marks ]**

**( b )** Solve the equation, $7^{2x} - 4(7^x) + 3 = 0$
giving your answers to 2 decimal places where appropriate.

**[ 6 marks ]**

**Question 12**

*A-Level Examination question from June 2010, paper C2, Q7 (Edexcel)*

**( a )** Given that

$$2\log_3(x-5)-\log_3(2x-13)=1$$

show that

$$x^2-16x+64=0$$

**[ 5 marks ]**

**( b )** Hence, or otherwise, solve

$$2\log_3(x-5)-\log_3(2x-13)=1$$

**[ 2 marks ]**

**Question 13**

*A-Level Examination Question from June 2002, paper P2, Q5 (Edexcel)*

**( a )** Given that

$$3 + 2\log_2 x = \log_2 y$$

show that

$$y = 8x^2$$

**[ 3 marks ]**

**( b )** Hence, or otherwise, find the roots $\alpha$ and $\beta$, where $\alpha < \beta$ of the equation,

$$3 + 2\log_2 x = \log_2 (14x - 3)$$

**[ 3 marks ]**

**( c )** Show that $\log_2 \alpha = -2$

**[ 1 mark ]**

**( d )** Calculate $\log_2 \beta$ giving your answer to 3 significant figures.

**[ 3 marks ]**

**Question 14**

*A-Level Examination Question from June 2009, paper C2, Q8 (Edexcel)*

**( a )** Find the value of $y$ such that

$$\log_2 y = -3$$

**[ 2 marks ]**

**( b )** Find the values of $x$ such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

**[ 5 marks ]**