

Lesson 7

A-Level Pure Mathematics : Year 1 Exponentials and Logarithms

7.1 Logarithms And Non-Linear Data

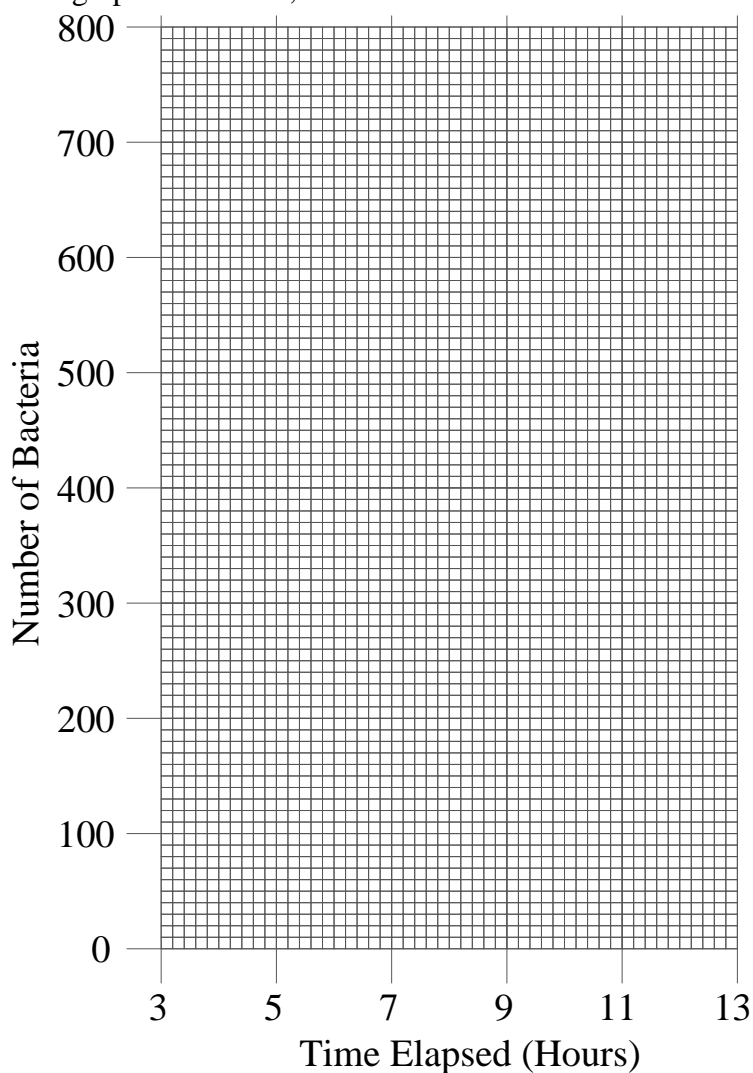
Logarithms are a surprisingly useful tool when it comes to looking at the data generated by experiments and observations in physics, chemistry, biology and economics.

7.2 Example

The table shows the number a bacteria, y , in a petri-dish after x hours have elapsed.

x hours	3	5	6	8	9	11	12	13
y bacteria	105	150	180	260	310	450	580	740

- (i) Plot a graph of the data;



[4 marks]

- (ii) By eye, draw in a line of best fit.
It should go through the point (Mean x , Mean y)

[2 marks]

- (iii) Determine the equation of your line of best fit.

[2 marks]

- (iv) Veronica has read in her biology textbook that bacteria, when unconstrained by food or space, exhibit exponential growth. She reads that to test for exponential growth the data is first coded using;

$$Y = \ln y \quad \text{and} \quad X = x$$

and then a plot of Y against X is made.

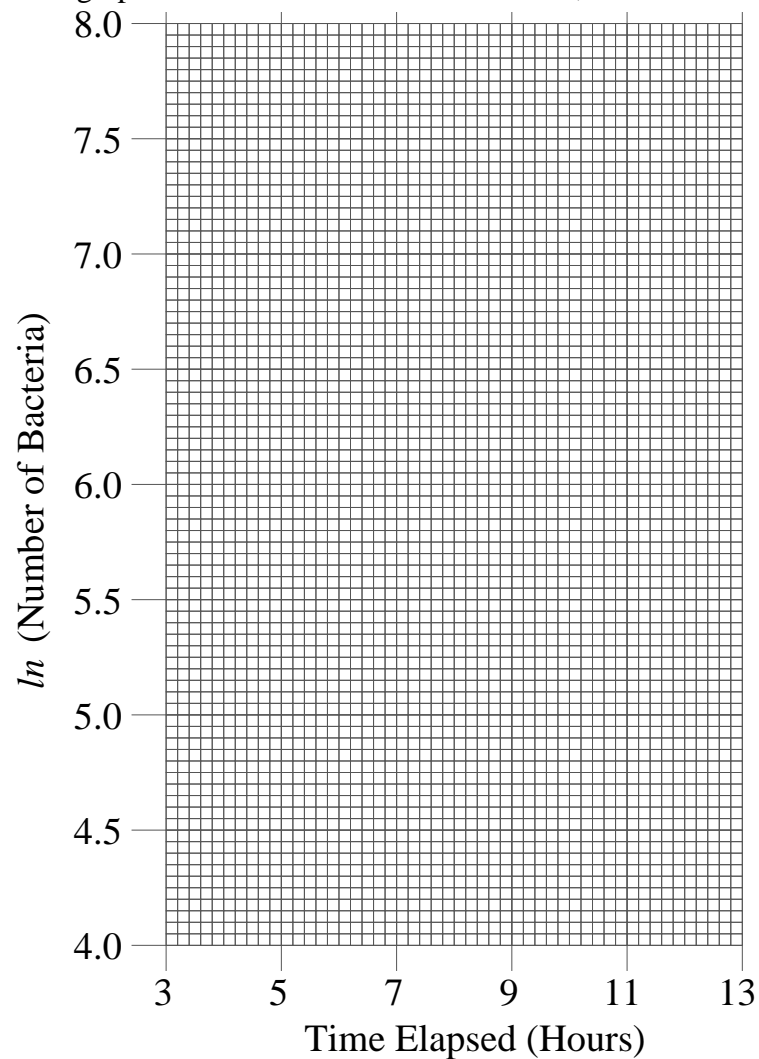
If a straight line relationship is then evident, the original (uncoded) growth data is indeed better modelled by an exponential curve.

Use this coding to complete the following table;

X	3			8				
$Y = \ln y$	4.65			5.56				

[2 marks]

- (v) Plot a graph of the coded bivariate data below;



[4 marks]

(vi) Is a straight line relationship evident in the graph of the coded data ?
[1 mark]

(vii) By eye, draw in the line of best fit.
[2 marks]

(viii) Determine the equation of your line of best fit.
it should go through the point (Mean X , Mean Y)

[2 marks]

(ix) How does your part (vii) answer confirm that an exponential curve is indeed a better fit for the uncoded data than a straight line ?

[1 mark]

(x) The general equation of an exponential curve is

$$y = a b^x$$

And the equation of the coded data straight line is

$$Y = A + B X$$

The values of a and b are found using the decoding

$$a = e^A \quad \text{and} \quad b = e^B$$

Write out the equation of the exponential curve of best fit in the form

$$y = a b^x$$

where a and b are numbers, correct to three significant figures that you have determined.

[3 marks]

(xi) Carefully plot the exponential curve of best fit onto your part (i) graph.
[1 mark]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk