9.1 The Power Function Model

There are many non-linear relationship models that are in common usage.
Here are six that are well known;

◇ The best ***quadratic*** curve: $y = a + bx + cx^2$

◇ The best ***logarithmic*** curve: $y = a + b \ln x$

◇ The best ***e powered*** curve: $y = a\, e^{bx}$

◇ The best (general) ***exponential*** curve: $y = a\, b^x$ (Lesson 8)

◇ The best ***power function*** curve: $y = a\, x^b$ (Lesson 9)

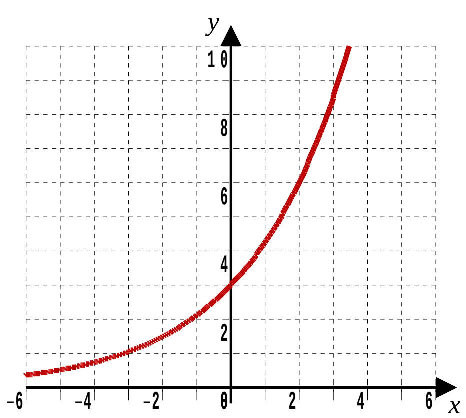
◇ The best ***simple reciprocal*** curve: $y = a + \dfrac{b}{x}$

There are many more and the adventurous may like to try building their own.

In other subjects, especially physics, chemistry, biology, economics and geography, you may want to explore some of these non-linear models if you are doing a project or investigation and need to fit a curve to your data.

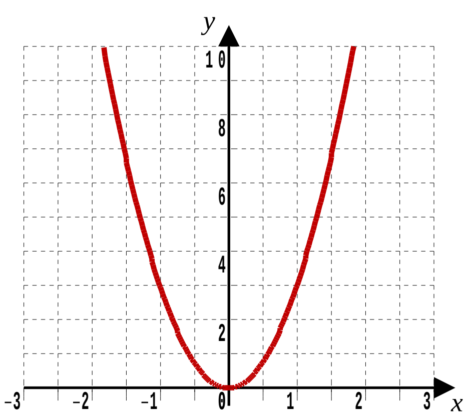
In this course one other Non-Linear Model is studied; the ***power function*** curve.

9.2 Spot The Difference



$y = 3 \times 2^x$

An ***exponential*** curve



$y = 3x^2$

A ***power function*** curve

9.3 The Theory Behind The Power Function Model

If

$$y = a\,x^b$$

then

$$\ln y = \ln\left(a\,x^b\right)$$

$$\ln y = \ln a + \ln x^b$$

$$\ln y = \ln a + b \ln x$$

which is in the form of the straight line

$$Y = A + BX$$

where $Y = \ln y$ $A = \ln a$ $B = b$ and $X = \ln x$

the first and last give the coding, reversing the middle two gives the decoding as;

$$a = e^A \quad \text{and} \quad b = B \qquad \square$$

Notice that the more complicated power function coding, compared with last lesson's exponential model, has yielded a simpler decoding.

9.4 Example

(i) Data are coded using

$$Y = \log y \quad \text{and} \quad X = x$$

and a curved graph results when the coded data is plotted as *Y* against *X*
What does this tell you about the relationship between the uncoded data, *y* and *x* ?

[1 mark]

(ii) The data are now coded using

$$Y = \log y \quad \text{and} \quad X = \log x$$

and a straight line graph results when the coded date is plotted, *Y* against *X*
What does this tell you about the relationship between the uncoded data, *y* and *x* ?

[2 marks]

(iii) Given that the line of best fit for the appropriately coded data is

$$Y = 1.2 + 0.4\,X$$

write down the relationship between *y* and *x* and find the values of the constants.

[3 marks]

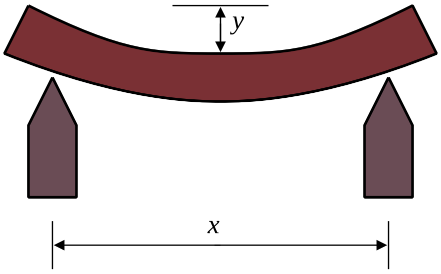
9.5 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 35

Question 1

An experiment is conducted to determine how the sag of a steel beam, y mm, varies with the distance between its supports, x m.



The data is coded using

$$Y = \log y \quad \text{and} \quad X = \log x$$

to give a linear relationship with line of best fit

$$Y = -0.947 + 2.84X$$

(i) Determine the power function curve of best fit.

It will be of the form, $y = a x^b$

where a and b are constants that you have to determine.

[2 marks]

(ii) Complete the following table,

x metres	2	4	6	8
y millimetres				

[2 marks]

(iii) Sketch the curve of best fit, marking on the value of any points where the curve passes through the coordinate axes.

[2 marks]

Question 2

A pendulum-swinging experiment is conducted to investigate how the average time of a swing, T, (measured in seconds) relates to the length, L, (measured in metres) of the pendulum.

The data is coded using

$$Y = \log T \qquad \text{and} \qquad X = \log L$$

to give a linear relationship with line of best fit

$$Y = 0.30 + 0.50\,X$$

(i) Determine the power function curve of best fit which will be of the form

$$T = a\,L^{b}$$

where a and b are constants that you have determined.

[2 marks]

(ii) Complete the following table,

L metres	0.1	0.2	0.5	1
T seconds				

[2 marks]

(iii) Sketch the curve of best fit, marking on the value of any points where the curve passes through the coordinate axes.

[2 marks]

Question 3

A scientist is modelling the number of people, N, who have fallen sick with a virus after t days. He codes his data using;

$$Y = \log N \qquad \text{and} \qquad X = t$$

and is pleased to see a linear relationship in the coded data.

His coded data's line of best fit passes through the end points (0, 2.4) and (6, 5.1)

(i) Determine the equation of the line of best fit for the coded data.

[2 marks]

(ii) Hence, write down the relationship between N and t that models the uncoded data. Include the values of any constants in the relationship.

[3 marks]

(iii) State the number of people that the model predicts will be sick after,

(a) 1 day **(b)** 4 days **(c)** 8 days

[3 marks]

(iv) Use your model to predict the number of sick people after 30 days.
Give one reason why this might be an overestimate.

[2 marks]

Question 4

The period of oscillation, P seconds, for bars of uniform material is thought to be proportional to some power of their length, L metres.

One set of measurements is given;

L	2	3	4	5	6
P	4.5	5.5	6.4	7.2	8.7

One result, however, has been incorrectly copied.

(i) Complete the following table;

$\ln L$		1.099			
$\ln P$				1.974	

[2 marks]

(ii) On the graph paper on the next page, plot the five points on a graph of $\ln P$ against $\ln L$

[2 marks]

(iii) Add an approximate straight line graph that goes through four of the five points; throw away the fifth error point.

[2 marks]

(iv) Determine the equation of your part (iii) line of best fit.

[3 marks]

(v) Now give the corresponding formula connecting P and L which should be of the form

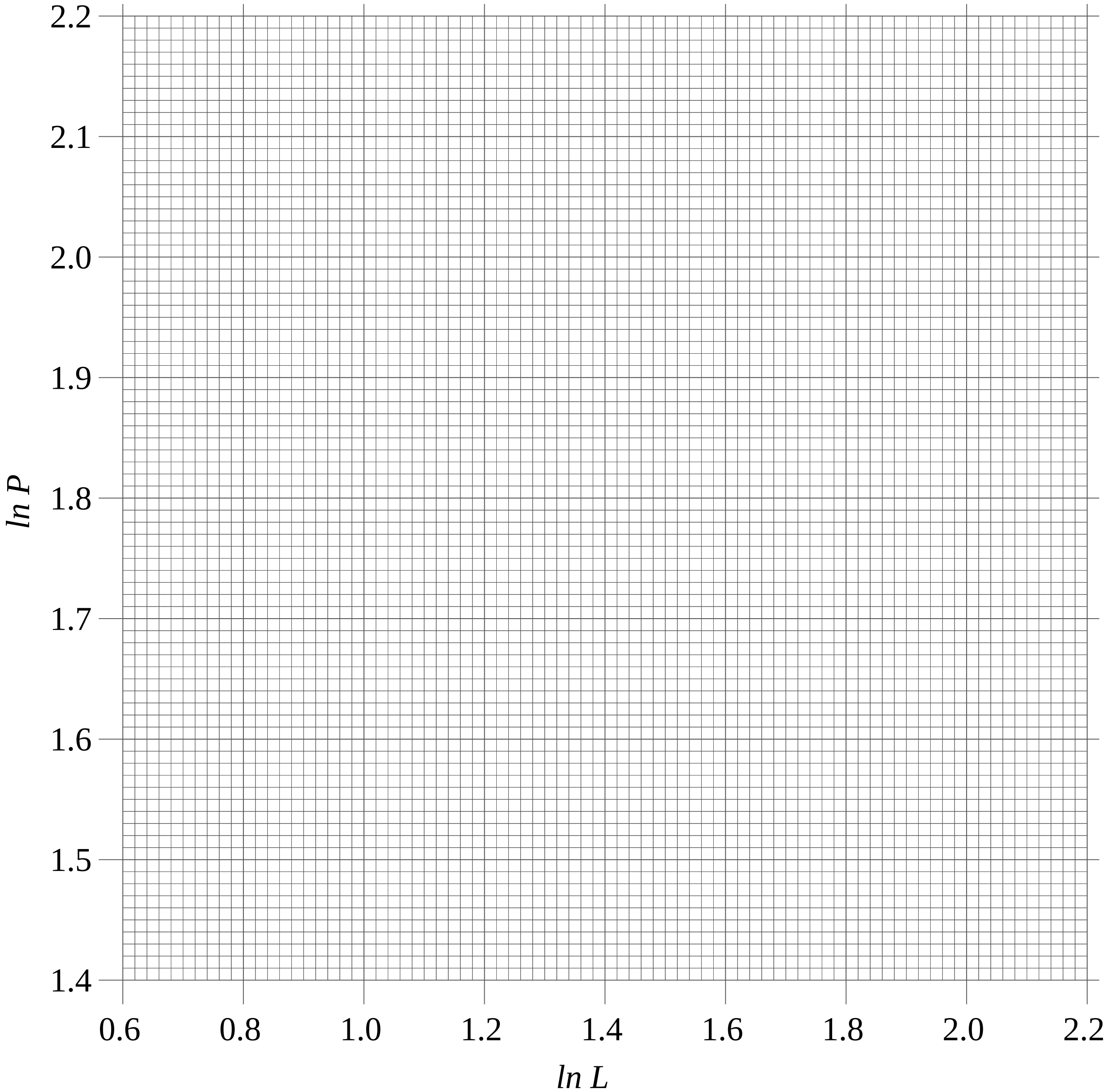
$$P = a L^b$$

where a and b are constants the values of which you have found.

[3 marks]

(vi) State what the incorrect result should have been.

[1 mark]


2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
ln P
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
ln L