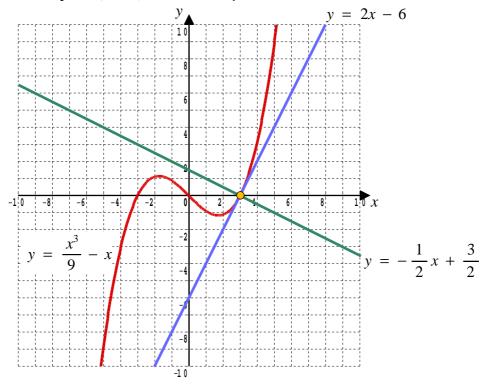
**Coordinate Geometry** 

#### 11.1 Normal from Curve

Previously, the curve with equation  $y = \frac{x^3}{9} - x$  was studied and the tangent

to it at the point (3, 0) found to be y = 2x - 6



There is a second line of interest, called the "normal" that is a right angles to the tangent at any specified point. At the point (3, 0) the normal to the curve

$$y = \frac{x^3}{9} - x$$
 turns out to be  $y = -\frac{1}{2}x + \frac{3}{2}$ .

Notice that the gradient of the tangent,  $m_t$ , and the gradient of the normal,  $m_n$  have the property of any pair of mutually perpendicular lines;  $m_t \times m_n = -1$  In other words, each is the sign changed reciprocal of the other.

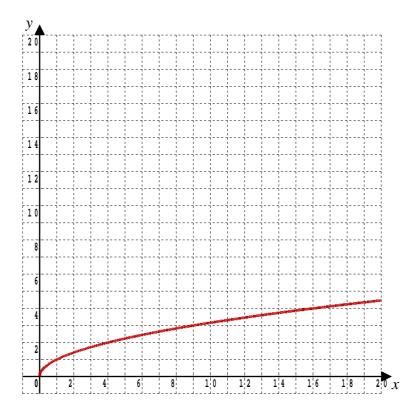
#### 11.2 Why the Normal is of Interest

Imagine the graph to be a road map and the curve a road on that map. A car moves along the road with constant speed. The tangent represents the direction a car moving along the road has at any moment. The normal represents the direction along which the force felt by a person in the car acts as it moves around each bend. Like the tangent the normal gives only a direction. It does not give the magnitude of the force; that depends on how sharply the road is bending and, indeed, on a straight piece of road the force along the normal has magnitude zero. The force along the normal is often referred to as centripetal force.

## 11.3 Example

The equation of a curve is  $y = \sqrt{x}$ 

- (i) Find the equation of the normal to this curve at the point where x = 4
- (ii) To the graph below add the part (i) normal.



Teaching Video: <a href="http://www.NumberWonder.co.uk/v9033/11.mp4">http://www.NumberWonder.co.uk/v9033/11.mp4</a>



### 11.4 Exercise

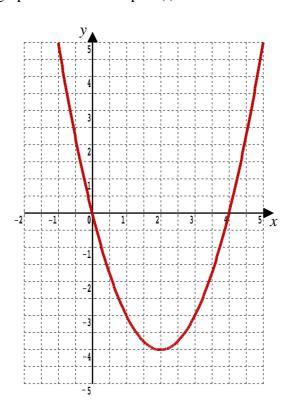
Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 52

# **Question 1**

The equation of a curve is  $y = x^2 - 4x$ 

- (i) Find the equation of the normal to this curve at the point where x = 4
- (ii) To the graph below add the part (i) normal



Additional Mathematics Examination Question from June 2009, Q2 (OCR) Find the equation of the normal to the curve

$$y = x^3 + 5x - 7$$

at the point (1, -1)

[ 5 marks ]

### **Question 3**

Additional Mathematics Examination Question from June 2019, Paper 1, Q3 (OCR) Find the equation of the normal to the curve

$$y = x^3 - 2x^2 + 2x + 4$$

at the point (2, 8)

Additional Mathematics Examination Question from June 2018, Q7 (OCR)

(i) Find the coordinates of the points where the line y = 7x - 9 cuts the curve  $y = x^2 + 2x - 5$ 

[ 4 marks ]

(ii) Determine whether the line is a normal to the curve at either of the points of intersection

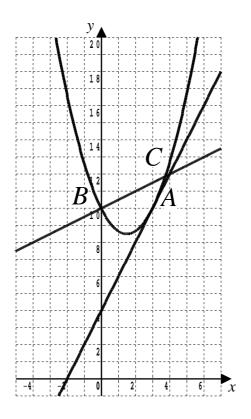
Additional Mathematics Examination Question from June 2014, Q10 (OCR)

(i) Find the coordinates of the point P on the curve  $y = 2x^2 + x - 5$  where the gradient of the curve is 5

[ 3 marks ]

(ii) Find the equation of the normal to the curve at the point P

Additional Mathematics Examination Question from June 2005, Q10 (OCR)



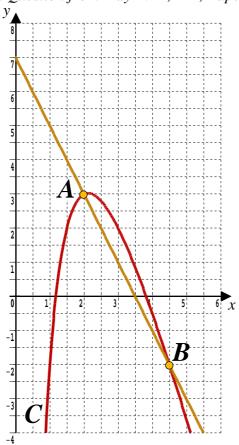
The curve shown has equation;

$$y = \frac{2}{3}x^2 - 2x + 10$$

(i) Find the equation of the tangent to the curve at A (3, 10)

( ii )	Show that the equation of the normal to the curve at $B(0, 10)$ is $2y - x = 20$	
( <b>iii</b> )	Find the coordinates of the point C where these two lines intersect	[ 3 marks ]
(m)	Find the coordinates of the point $C$ where these two lines intersect	
( iv )	Calculate the length $BC$	[ 3 marks ]
		[ 2 marks ]

A-Level Examination Question from May 2014, IAL, Paper C1(R), Q11 (Edexcel)



The sketch is of part of the curve C with equation  $y = 20 - 4x - \frac{18}{x}$ , x > 0

Point A lies on C and has an x coordinate equal to 2

(a) Show that the equation of the normal to C at A is y = -2x + 7

The normal to C at A meets C again at the point B  ( <b>b</b> ) Use algebra to find the coordinates of B						
(~)	222 mgrota to 1ma are coordinates					
			[5 marks]			