Coordinate Geometry

2.1 Line Segments

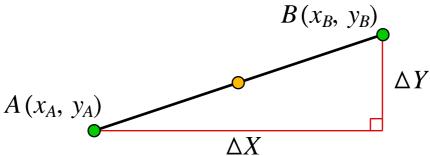
A line segment is simply a piece of straight line between two endpoints.

The length of a line segment is found by using the theorem of Pythagoras.

The coordinates of the midpoint of a line segments are found by taking the average (the mean) of the x parts of the two endpoints, to give the x part of the midpoint, and, separately, taking the average (the mean) of the y parts of the two endpoints to give the y part of the midpoint.

These comments are formalised in the following theorem;

The Length and Midpoint of a Line Segment



A line segment with endpoints $A(x_A, y_A)$ and $B(x_B, y_B)$ has a length given by;

$$|AB| = \sqrt{(\Delta X)^2 + (\Delta Y)^2}$$

where $\Delta X = x_B - x_A$ and $\Delta Y = y_B - y_A$

Furthermore,

Midpoint AB =
$$\left(\frac{x_B + x_A}{2}, \frac{y_B + y_A}{2}\right)$$

The $\triangle X$ and $\triangle Y$ are the same as that used to calculate gradient, $m = \frac{\triangle Y}{\triangle X}$

In three dimensions with endpoints $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$ the theorem is only marginally more complicated;

$$|AB| = \sqrt{(\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2}$$
where $\Delta X = x_B - x_A$, $\Delta Y = y_B - y_A$ and $\Delta Z = z_B - z_A$

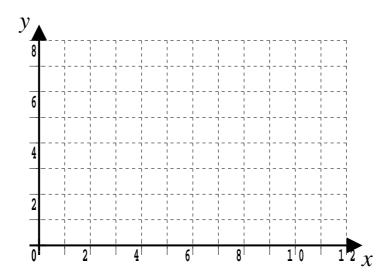
$$Midpoint AB = \left(\frac{x_B + x_A}{2}, \frac{y_B + y_A}{2}, \frac{z_B + z_A}{2}\right)$$

2.2 Example

The graph of this function is a line segment.

$$f(x) = \frac{1}{2}x + 1, \quad x \in \mathbb{R}, \ 2 \le x \le 10$$

Sketch the line segment on the grid below, and determine its length and midpoint using the theorem, "The Length and Midpoint of a Line Segment"



Teaching Video: http://www.NumberWonder.co.uk/v9033/2.mp4



2.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

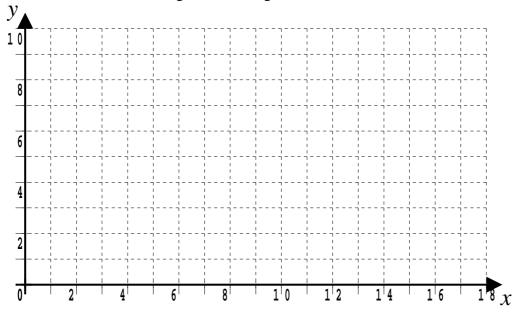
Marks Available: 55

Question 1

The graph of this function is a line segment.

$$g(x) = -\frac{1}{3}x + 8, \quad x \in \mathbb{R}, \ 3 \le x \le 15$$

(i) Sketch the line segment on the grid below



[3 marks]

(ii) Use the theorem "The Length and Midpoint of a Line Segment" to calculate the exact length of the line segment.

[2 marks]

(iii) Use the theorem "The Length and Midpoint of a Line Segment" to determine midpoint of the line segment.

[2 marks]

Question 2

Without drawing a graph, and showing your working, determine the exact distance between the two points A(-15, -2) and B(9, 5)

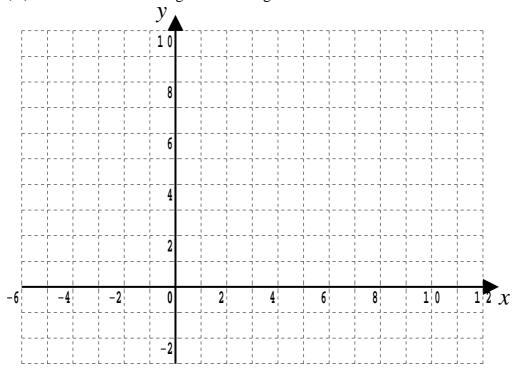
[3 marks]

Question 3

The graph of this function is a line segment.

$$h(x) = \frac{2}{3}x + 1, \quad x \in \mathbb{R}, \quad -3 \le x \le 9$$

(i) Sketch the line segment on the grid below



[3 marks]

(ii) Use the theorem "The Length and Midpoint of a Line Segment" to calculate the exact length of the line segment.

[2 marks]

(iii) Use the theorem "The Length and Midpoint of a Line Segment" to determine midpoint of the line segment.

[2 marks]

Question 4

A straight line has equation 4y - 12x + 3 = 0

Write this equation in the form y = mx + c

[2 marks]

Question 5

IGCSE Examination Question from January 2020, Paper 1H, Q1 (Edexcel)

The point A has coordinates (5, -4)

The point B has coordinates (13, 1)

(a) Work out the coordinates of the midpoint of AB

[2 marks]

Line L has equation y = 2 - 3x

(**b**) Write down the gradient of line L

[1 mark]

(c) Does the point with coordinates (100, -302) lie on L? You must give a reason for your answer

[1 mark]

Question 6

IGCSE Examination Question from January 2018, Paper 3H, Q10 (Edexcel)

The straight line L is parallel to the line with equation 2y + 8x = 5

L passes through the point with coordinates (2,3)

Find an equation for L

	and Mathematics Examination Question from June 2017, Q4 (OCR, FSMQ) ordinates of A and B are $(1, 5)$ and $(-3, 7)$ respectively Calculate the exact length of AB	
(ii)	Find the coordinates of the midpoint of AB	
	[1 mark]	
	on 8 nal Mathematics Examination Question from June 2009, Q3 (OCR, FSMQ) point $(1, 5)$ and C is the point $(3, p)$	
(i)	Find the equation of the line through A which is parallel to $2x + 5y = 7$	
(ii)	[2 marks] This line also passes through the point C .	

Find the value of p

Question 9

IGCSE Examination Question from June 2017, Paper 4H, Q13 (Edexcel) Here are the equations of four straight lines,

Line **A**
$$y = 2x + 3$$

Line **B**
$$2y = 6 - 3x$$

Line **C**
$$4x - 2y = 3$$

$$Line D y = 3 - 2x$$

Two of these lines are parallel.

(a) Which two lines?

[2 marks]

Line **L** has a gradient of $-\frac{5}{2}$ and passes through the point with coordinates (1, 3)

(b) Find an equation of L

Give your answer in the form ax + by = c where a, b and c are integers

[3 marks]

Question 10

A-Level Examination Question from January 2008, C1, Q4.

The point A(-6, 4) and the point B(8, -3) lie on the line L.

(a) Find an equation for L in the form ax + by + c = 0, where a, b and c are integers.

[4 marks]

(**b**) Find the distance *AB*, giving your answer in the form $k\sqrt{5}$, where *k* is an integer.

[3 marks]

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Question 11 A-Level Examination Question from May 2010, C1, Q8 Find an equation of the line joining A(7, 4) and B(2, 0), giving your answer in the form ax + by + c = 0, where a, b and c are integers [3 marks] **(b)** Find the length of AB, leaving your answer in surd form [2 marks] The point C has coordinates (2, t), where t > 0, and AC = AB(c) Find the value of t [1 mark] (d) Find the area of triangle *ABC*

[2 marks]