Lesson 4

A-Level Pure Mathematics, Year 1
Additional Mathematics
GCSE
Coordinate Geometry

4.1 Lines at Right Angles

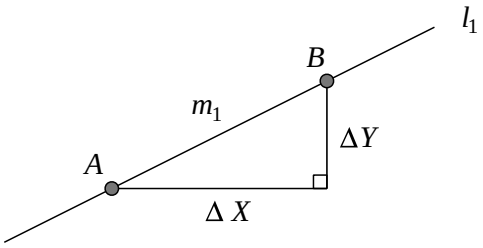
The Perpendicular Lines Theorem (Version 1)

If the gradient of the line l_1 is m_1 and the gradient of the line l_2 is m_2 then the lines l_1 and l_2 are perpendicular if and only if

$$m_1 \times m_2 = -1$$

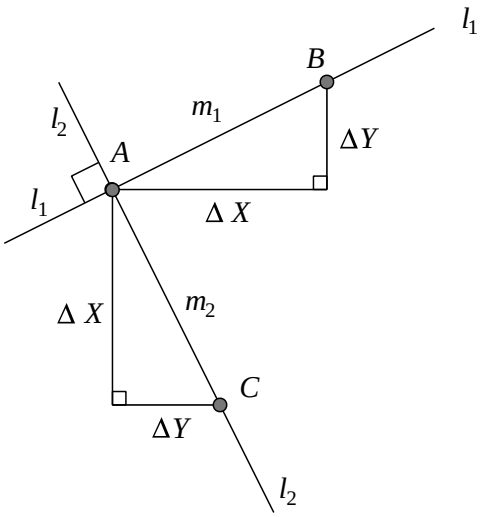
Proof

Consider two points, A and B on the line l_1 which has gradient m_1



Clearly, $m_1 = \dfrac{\Delta Y}{\Delta X}$

Now, consider a rotation of $-90°$ about the point A which gives a line l_2 with gradient m_2 which is perpendicular to l_1.



Clearly, $m_2 = -\dfrac{\Delta X}{\Delta Y}$

Observe that, $m_1 \times m_2 = \dfrac{\Delta Y}{\Delta X} \times -\dfrac{\Delta X}{\Delta Y}$

$= -1$ □

In many questions, the gradient of a first line will be known and the gradient of a second, perpendicular to the first, sought.
In consequense the following version of the theorem is often of more use;

The Perpendicular Lines Theorem (Version 2)

Given a line l_1 with gradient m_1 then the gradient m_2 of any perpendicular line l_2 is the ***sign changed reciprocal*** of m_1.

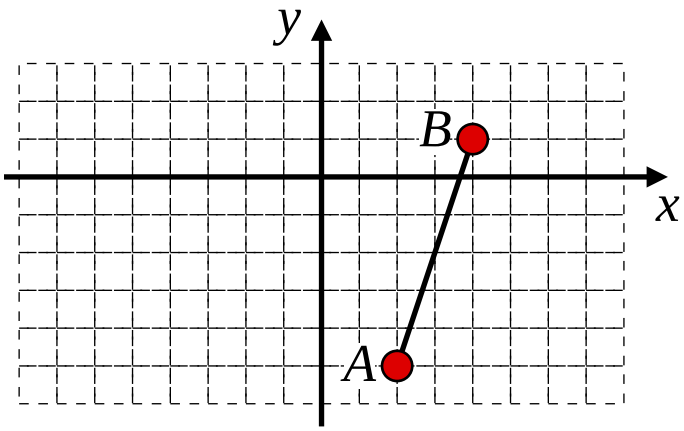
$$\text{That is, } m_2 = -\frac{1}{m_1}$$

4.2 Example

Find the equation of the perpendicular bisector of the line segment AB where A is $(2, -5)$ and B is $(4, 1)$

(i) Give your answer in the form $y = mx + c$

(ii) Illustrate your answer with a sketch graph.



Teaching Video : http://www.NumberWonder.co.uk/v9033/4.mp4

[4 marks]

4.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Question 1

A line, L, has equation

$$y = \frac{2}{3}x + \frac{1}{3}$$

(i) What is the gradient of L ?

[1 mark]

(ii) What would be the gradient of a line, perpendicular to L ?

[1 mark]

Question 2

A line has equation $2x + 5y - 4 = 0$

(i) Write this line's equation in the form $y = mx + c$

[1 marks]

(ii) Hence state the gradient of the line $2x + 5y - 4 = 0$

[1 mark]

(iii) What is the gradient of a line, perpendicular to $2x + 5y - 4 = 0$?

[1 mark]

Question 3

A line has equation $5x - 3y - 2 = 0$

(i) What is the gradient of this line ?

[2 marks]

(ii) What is the gradient of a line perpendicular to $5x - 3y - 2 = 0$?

[1 mark]

Question 4

Additional Mathematics Examination Question from June 2015, Q1, (OCR, FSMQ)

Find the equation of the line which is perpendicular to the line $2x + 3y = 5$ and which passes through the point $(3, 4)$

[3 marks]

Question 5

A-Level Examination question from May 2011, C1, Q3 (Edexcel)

The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively.
The line l is perpendicular to PQ and passes through the mid-point of PQ.
Find an equation for l, giving your answer in the form $ax + by + c = 0$,
where a, b and c are integers.

[5 marks]

Question 6

A-Level Examination Question from January 2010, C1, Q3 (Edexcel)

The line l_1 has equation $3x + 5y - 2 = 0$

(a) Find the gradient of l_1

[2 marks]

The line l_2 is perpendicular to l_1 and passes through the point (3, 1)

(b) Find the equation of l_2 in the form $y = mx + c$, where m and c are constants

[3 marks]

Question 7

A-Level Examination Question from January 2006, C1, Q3 (Edexcel)

The line L has equation $y = 5 - 2x$

(a) Show that the point $P(3, -1)$ lies on L

[1 mark]

(b) Find an equation of the line perpendicular to L, which passes through P. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[4 marks]

Question 8

Additional Mathematics Examination Question from June 2014, Q8 (OCR)

Four points have coordinates $A(-5, -1)$, $B(0, 4)$, $C(7, 3)$ and $D(2, -2)$

(i) Using gradients of lines, prove that *ABCD* is a parallelogram

[2 marks]

(ii) Using lengths of lines, prove that *ABCD* is a rhombus

[2 marks]

(iii) Prove that *ABCD* is not a square

[2 marks]

Question 9

A-Level Examination Question from January 2011, C1, Q9 (Edexcel)

The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point $A(1, 4)$ lies on L_1 find,

(a) the value of k,

[1 mark]

(b) the gradient of L_1

[2 marks]

The line L_2 passes through A and is perpendicular to L_1

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[4 marks]

The line L_2 crosses the x-axis at the point B

(d) Find the coordinates of B

[2 marks]

(e) Find the exact length of AB

[2 marks]

Question 10

A-Level Examination Question from May 2007, C1, Q11 (Edexcel)

The line l_1 has equation $y = 3x + 2$,

and the line l_2 has equation $3x + 2y - 8 = 0$

(a) Find the gradient of the line l_2

[2 marks]

The point of intersection of l_1 and l_2 is P

(b) Find the coordinates of P

[3 marks]

The lines l_1 and l_2 cross the line $y = 1$ at the points A and B respectively.

(c) Find the area of triangle ABP

[4 marks]