Coordinate Geometry

6.1 The Equation of a Circle

In the work of straight lines, a geometric object was analysed using algebra. Extensive use was made of the equation y = mx + c and an understanding of how the various parts of that equation related to the particular straight line under consideration; that the value of m gave information about gradient, and that the value of c gave information about where the line intercepted the y-axis.

A similar approach is taken with circles.

The Equation of a Circle

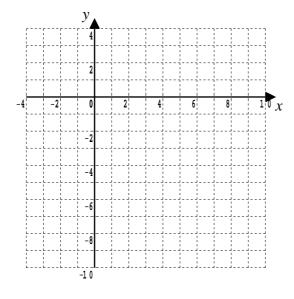
$$(x-a)^2 + (y-b)^2 = r^2$$

represents a circle with centre (a, b) and radius r

6.2 Example

For the circle with equation $(x-4)^2 + (y+3)^2 = 25$

- (i) Write down the coordinates of the circle's centre and state its radius.
- (ii) Prove that the circle passes through the origin.
- (iii) Sketch the circle, using the NEWS method.



[2, 2, 2 marks]

Teaching Video: http://www.NumberWonder.co.uk/v9033/6.mp4



The video walks through the above example

6.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 60

Question 1

$$(x-16)^2 + (y-9)^2 = 100$$

State the radius and the centre of this circle

[2 marks]

Question 2

$$(x + 25)^2 + (y + 121)^2 = 17^2$$

State the radius and the centre of this circle

[2 marks]

Question 3

$$(x-12)^2 + (y+5)^2 = 400$$

(i) State the radius and the centre of this circle, C

[2 marks]

(ii) What is the distance between the centre of the circle, C, and the origin, O?

[2 marks]

Question 4

Write down the equation of a circle, centre (2, 8) and radius 7

[2 marks]

Question 5

Write down the equation of a circle, centre (-2.1, 4.8) and radius 3.6

[2 marks]

Question 6

Write down the equation of a circle, centre (-1, -5) and radius $\sqrt{13}$

[2 marks]

Question 7

Additional Mathematics Examination Question from June 2006, Q4 (OCR)

(i) Find the distance between the points (2, 3) and (7, 9)

[2 marks]

(ii) Hence find the equation of the circle with centre (2, 3) and passing through the point (7, 9)

[2 marks]

Question 8

Additional Mathematics Examination Question from June 2009, Q4, (OCR) AB is a diameter of a circle, where A is (1, 1) and B is (5, 3) Find,

(i) the exact length of AB

[2 marks]

(ii) the coordinates of the midpoint of AB

[1 mark]

(iii) the equation of the circle

[3 marks]

Question 9

Additional Mathematics Examination Question from June 2011, Q1 (OCR) Determine whether the point (5, 2) lies inside or outside the circle whose equation is $x^2 + y^2 = 30$

You must show your working.

Question 10

Consider the circle with equation $(x - 10)^2 + (y - 8)^2 = 100$

(i) Write down the coordinates of the circle's centre

[1 mark]

(ii) What is the radius of the circle?

[1 mark]

(iii) Prove that the point (18, 14) is on the circle

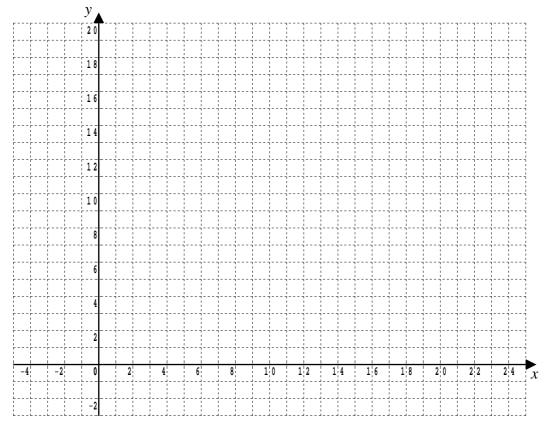
[2 marks]

(iv) Find the coordinates of the two points where the circle crosses the x-axis

[2 marks]

- (v) Sketch the circle by first plotting points
 - with the help of the NEWS method
 - using the answers to parts (iii) and (iv)

and making use of the circle's horizontal and vertical lines of mirror symmetry



[3 marks]

Question 11 Additional Mathematics Examination Question from May 2012, Q10 (OCR) $A(1, 10), B(8, 9)$ and $C(7, 2)$ are three points (i) Find the coordinates of the midpoint, M , of AC		
		[1 mark]
(ii)	Find the equation of the circle with AC as diameter	
		[4 marks]
(iii)	Show that <i>B</i> lies on this circle	
		[1 mark]
(iv)	Prove that AM and BM are perpendicular	[I mark]

BD is a diameter of this circle. Find the coordinates of D

(v)

[3 marks]

[3 marks]

Question 12

Additional Mathematics Examination Question from June 2013, Q11 (OCR)

A circle has equation $(x-2)^2 + y^2 = 100$

(a) Write down the radius and the coordinates of the centre, C, of this circle

[2 marks]

The line y = 2x + 6 cuts the circle at two points, A and B

- **(b)** Find
 - (i) the coordinates of A and B

[5 marks]

(ii) the midpoint, M, of AB

[1 mark]

(iii) the length AB

[2 marks]

(\mathbf{c}) Hence find the distance of the centre of the circle from the line AB

[2 marks]