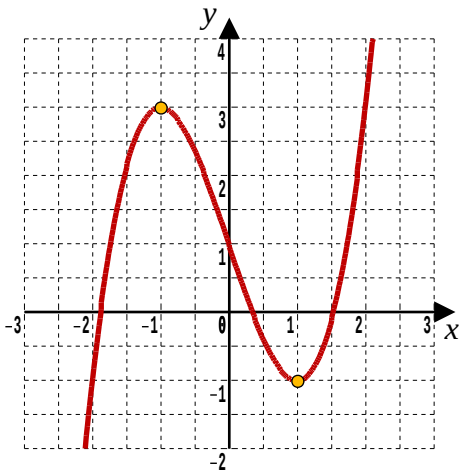
Lesson 3

GCSE Mathematics
Differentiation I

3.1 Turning Points

A turning point is a point of a graph where the gradient "turns through zero".

For example, the graph of $y = x^3 - 3x + 1$ has two turning points.



Remembering that a graph is read from left to right, at $(-1,\ 3)$ the gradient turns from being positive (before $x = -1$) through zero (at $x = -1$) to being negative (after $x = -1$). At $(1, -1)$ the gradient turns from being negative (before $x = 1$) through zero (at $x = 1$) to being positive (after $x = 1$).
The graph thus has two turning points, one at $(-1,\ 3)$ and the other at $(1, -1)$.
How can these be found without drawing a graph ?

Teaching Video : http://www.NumberWonder.co.uk/v9036/3.mp4

After watching the video,
set out the method of
solution here

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[6 marks]

3.2 Exercise

Marks Available : 50

Question 1

For each of these equations, determine $\frac{dy}{dx}$

(i) $y = 7x^5$ $\frac{dy}{dx} =$

(ii) $y = 8x + 1.5$ $\frac{dy}{dx} =$

(iii) $y = 5x^8 + 17x - 11$ $\frac{dy}{dx} =$

[6 marks]

Question 2

$y = 5x^2 - 30x$

(a) Find $\frac{dy}{dx}$

[2 marks]

(b) Find the coordinates of the turning point by solving the equation;

$$\frac{dy}{dx} = 0$$

Show your working clearly.

[5 marks]

Question 3

Differentiate each of the following,

(i) $y = 24x^2 - 12x^4$

(ii) $y = 13$

(iii) $y = \frac{5}{x^3}$

[6 marks]

Question 4

$y = 4x^2 + 16x + 21$

(a) Find $\frac{dy}{dx}$

[2 marks]

(b) Find the coordinates of the turning point by solving the equation;

$$\frac{dy}{dx} = 0$$

Show your working clearly.

[5 marks]

Question 5

By first expanding the brackets, find the derivative of,

$$y = x^3\left(4x^8 - 7x\right)$$

[4 marks]

Question 6

$y = x^3 + 9x^2 + 15x$

(a) Find $\frac{dy}{dx}$

[2 marks]

The curve with equation $y = x^3 + 9x^2 + 15x$ has two turning points.

(b) Work out the coordinates of these two turning points.
Show your working clearly.

[6 marks]

Question 7

GCSE Question, 9th January 2017, Paper 3H, Q18

The curve with equation

$$y = 10x^2 + 9x + 5$$

has a minimum at point *A*

Find the coordinates of *A.*
Show your working clearly.

HINT : The minimum is a turning point.

[4 marks]

Question 8

GCSE Question, 4th June 2015, Paper 4H, Q20

$y = x^3 + 6x^2 + 5$

(a) Find $\frac{dy}{dx}$

[2 marks]

The curve with equation $y = x^3 + 6x^2 + 5$ has two turning points.

(b) Work out the coordinates of these two turning points.
Show your working clearly.

[6 marks]