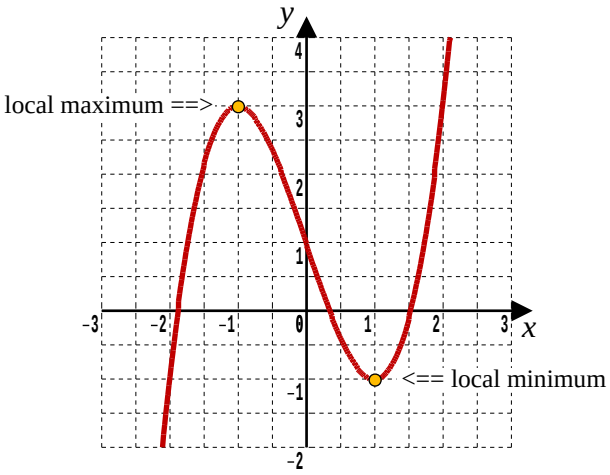
**Lesson 6**

**GCSE Mathematics**
**Differentiation I**

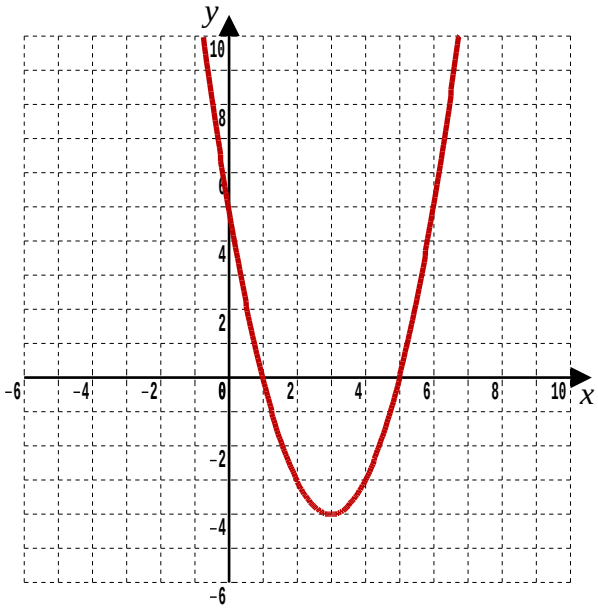
## 6.1 Local Minimum & Local Maximum

Differentiation is used to find the optimal solutions to problems.
On a graph, such 'best' solutions are often found where there is either a **local maximum** or a **local minimum**.




## 6.2 The Graphical Method

The graph below is of the equation $y = x^2 - 6x + 5$




By looking at the graph, write down the integer coordinates of the local minimum.

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**[ 1 mark ]**

As mathematicians, we don't want to have to go to the bother of plotting the graph to find this important point.

### 6.3 The Mathematical Method

Teaching Video : http://www.NumberWonder.co.uk/v9036/6.mp4

The teaching video will talk you through the following method of finding all local minima and local maxima of a function.

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**Finding Local Minima and Local Maxima**

STEP 1 : Differentiate the **POINTS** equation to get its **GRADIENT** equation.
STEP 2 : Set the **GRADIENT** equation equal to zero and solve.
STEP 3 : Put the solution(s) from STEP 2 back into the **POINTS** equation.

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**Example**

Find all local maxima and minima (if any) on the curve with equation;

$$y = x^2 - 6x + 5$$

**[ 3 marks ]**

### 6.4 Exercise

Marks Available : 50

**Question 1**

Find the coordinates of the **local minimum** point on the following quadratic curve;

$$y = x^2 - 8x + 9$$

**[ 3 marks ]**

**Question 2**

Find the coordinates of the **local maximum** point on the following parabola,

$$y = 6x + 14 - x^2$$

**[ 3 marks ]**

**Question 3**

Find the coordinates of the **local minimum** point on the following parabola,

$$y = 2x^2 - 20x + 52$$

**[ 3 marks ]**

**Question 4**

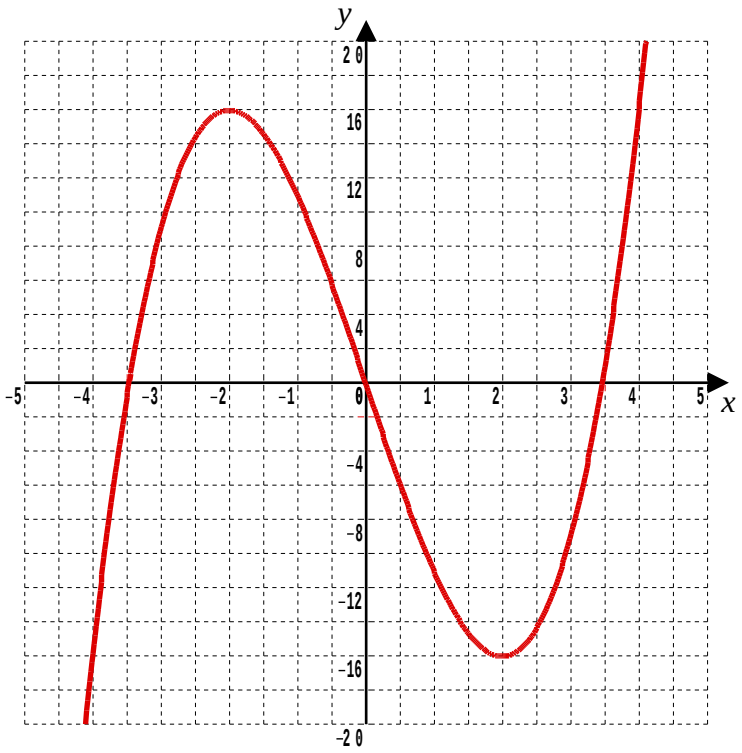
Find the coordinates of the **local maximum** point on the following parabola

$$y = 12x - 7 - 3x^2$$

**[ 3 marks ]**

**Question 5**

Consider the equation, $y = x^3 - 12x$




**( a )** From looking at the curve,

**( i )** write down the coordinates of the **local maximum** point.

**[ 1 mark ]**

**( ii )** write down the coordinates of the **local minimum** point.

**[ 1 mark ]**

( **b )** Use the mathematical method to obtain the same answers.

**[ 4 marks ]**

**Question 6**

Find the coordinates of any **local minimum** or **local maximum** point on;

**( i )** $y = x^3 - 27x$

**[ 4 marks ]**

**( ii )** $y = (x + 7)(x + 1)$

**[ 4 marks ]**

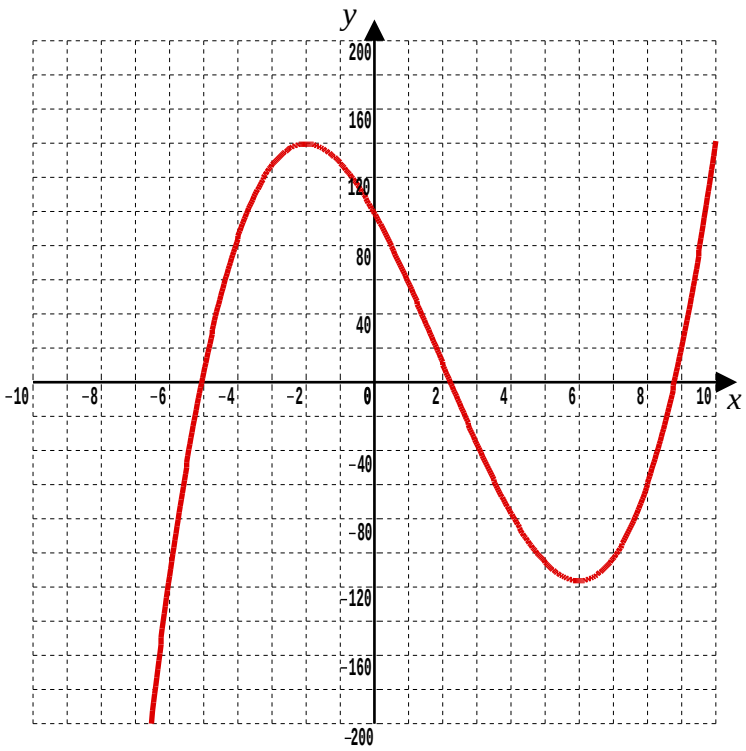
**( iii )** $y = x^4 - 256x$

**[ 4 marks ]**

**Question 7**

Use mathematics to find the local minimum and local maximum of the curve,

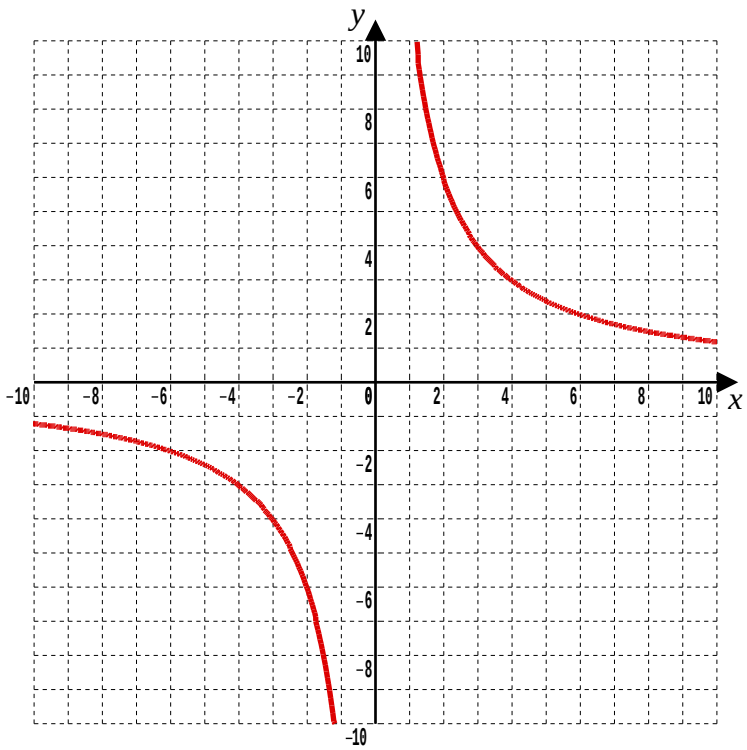
$$y = x^3 - 6x^2 - 36x + 100$$




**[ 6 marks ]**

**Question 8**

The graph is of the "inverse proportion" function $f(x) = \frac{12}{x}$




**( i )** Write down the gradient function, $f'(x)$

**[ 2 marks ]**

**( ii )** Write down the bend detector function, $f''(x)$

**[ 2 marks ]**

**( iii )** Use the appropriate function to find the point on this curve where $x = 2$

**[ 2 marks ]**

**( iv )** Use the appropriate function to find the gradient of this curve when $x = 2$

**[ 2 marks ]**

**( v )** Determine if the curve is bending anticlockwise or clockwise when $x = 2$

**[ 2 marks ]**

**Question 9**

The curve $y = x^3 + 12x$ has no turning points

Show that this is the case by trying to find them via the mathematical method.

What goes “wrong” ?

**[ 4 marks ]**