Lesson 5

A-Level Pure Mathematics : Year 2
Trigonometric Identities

5.1 The Reciprocal Trigonometric Functions

The reciprocal trig functions are;

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

A further two new identities, derived from an old favourite immediately follow;

$\cos^2\theta + \sin^2\theta = 1$ This is the old favourite which yields,

$1 + \tan^2\theta = \sec^2\theta$ upon dividing through the old favourite by $\cos^2\theta$, and,

$\cot^2\theta + 1 = \csc^2\theta$ upon dividing through the old favourite by $\sin^2\theta$

5.2 Example

Prove that, $\csc\theta\left(\cos\theta\cot\theta + \sin\theta\right) - 1 = \cot^2\theta$

Teaching Video : http://www.NumberWonder.co.uk/v9040/5.mp4

After watching the Teaching Video, write out the proof,

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[3 marks]

5.3 Proofs Strategy

When tackling proof questions involving $sec\ \theta$, $csc\ \theta$ and $cot\ \theta$;

- If no powers are involved, change $sec\ \theta$, $csc\ \theta$ and $cot\ \theta$ into the more familiar trigonometric ratios using

$$sec\ \theta = \frac{1}{cos\ \theta}, \qquad csc\ \theta = \frac{1}{sin\ \theta} \qquad \text{and} \qquad cot\ \theta = \frac{1}{tan\ \theta}$$

- As soon as powers occur, especially a square, try to make use of any of,

$$cos^2\ \theta + sin^2\ \theta = 1$$
$$1 + tan^2\theta = sec^2\ \theta$$
$$cot^2\ \theta + 1 = csc^2\ \theta$$

- It follows from the fact that $tan\ \theta = \frac{sin\ \theta}{cos\ \theta}$ that $cot\ \theta = \frac{cos\ \theta}{sin\ \theta}$

5.4 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 40

Question 1

Prove that, $sin\ \theta\ (\ csc\ \theta - sin\ \theta\ ) + cos\ \theta\ (\ sec\ \theta - cos\ \theta\ ) = 1$

[3 marks]

Question 2

Prove that, $\dfrac{\sin\theta\ (\sin\theta - \csc\theta)}{\cos\theta\ (\cos\theta - \sec\theta)} = \cot^2\theta$

[3 marks]

Question 3

Prove that, $\dfrac{(1 - \sin\theta)\ (1 + \sin\theta)}{(1 - \cos\theta)\ (1 + \cos\theta)} = \cot^2\theta$

[3 marks]

Question 4

Prove that, $sec^2\,\theta - tan^2\,\theta + csc^2\,\theta - cot^2\,\theta = 2$

[3 marks]

Question 5

Prove that, $sec\,\theta\,(\,cos\,\theta + sin\,\theta\,tan\,\theta\,) = sec^2\,\theta$

[3 marks]

Question 6

Prove that, $\dfrac{1}{\csc\theta\ (\cos\theta\cot\theta\ +\ \sin\theta)} = \sin^2\theta$

[3 marks]

Question 7

Prove that, $\dfrac{1}{(\tan^2\theta\ +\ 1)} + \dfrac{1}{(\cot^2\theta\ +\ 1)} = 1$

[3 marks]

Question 8

Prove that, $(\sec^2\theta - 1)(\csc^2\theta - 1) = 1$

[3 marks]

Question 9

Prove that, $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

[3 marks]

Question 10

Prove that, $\frac{\cos\theta}{\sqrt{1 + \tan^2\theta}} + \frac{\sin\theta}{\sqrt{1 + \cot^2\theta}} = 1$

[3 marks]

Question 11

Prove this identity, $\cot\theta - \tan\theta = 2\cot 2\theta$

[5 marks]

Question 12

Prove this identity, $\cot\theta + \tan\theta = 2\csc 2\theta$

[5 marks]