5.1 Transformation Of Graphs (Part 2)

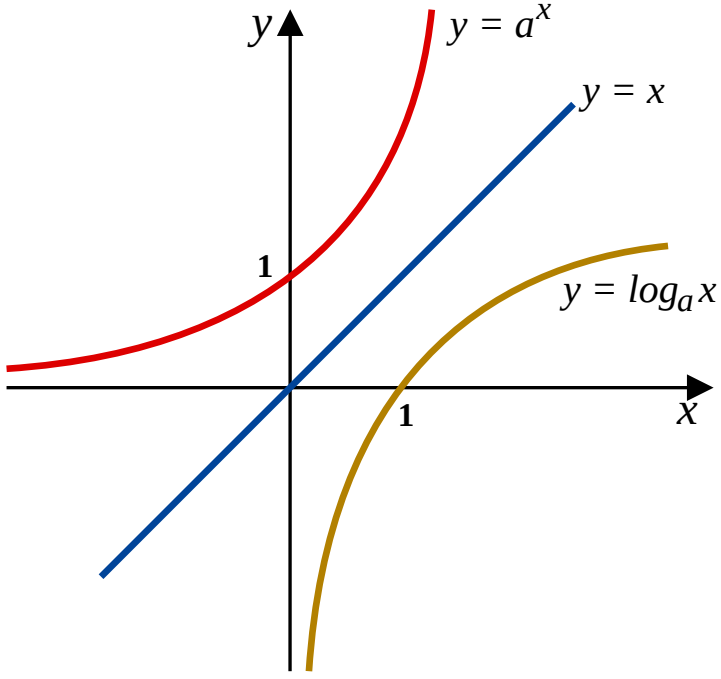
Replace all occurrences of...	...with	Effect on graph (Transformation)
x	$(x - a)$	Translation $\begin{pmatrix} a \\ 0 \end{pmatrix}$
y	$(y - b)$	Translation $\begin{pmatrix} 0 \\ b \end{pmatrix}$
x	$(-x)$	Reflection in the y-axis
y	$(-y)$	Reflection in the x-axis
x	(cx)	Stretch parallel to the x-axis with scale factor $\frac{1}{c}$
y	(dy)	Stretch parallel to the y-axis with scale factor $\frac{1}{d}$

- ◇ Replacing all occurrences of x with y AND all occurrences of y with x causes reflection in the line $y = x$.
- ◇ Reflecting the graph of a one-to-one function in the line $y = x$ gives the graph of the inverse function.

5.2 Example #1

If $f(x) = a^x$ then $f^{-1}(x) = \log_a x, \quad x > 0$

When plotted on a common graph each is a reflection of the other in the line $y = x$



5.3 Example #2

Sketch on separate diagrams the graph of the following four related equations, each time, stating the range of the corresponding function.

(i) $y = (x - 3)^2 - 4$ **(ii)** $y = \left| (x - 3)^2 - 4 \right|$

(iii) $y + 2 = \left| (x - 3)^2 - 4 \right|$ **(iv)** $2 - y = \left| (x - 3)^2 - 4 \right|$

[3, 3, 3, 3 marks]

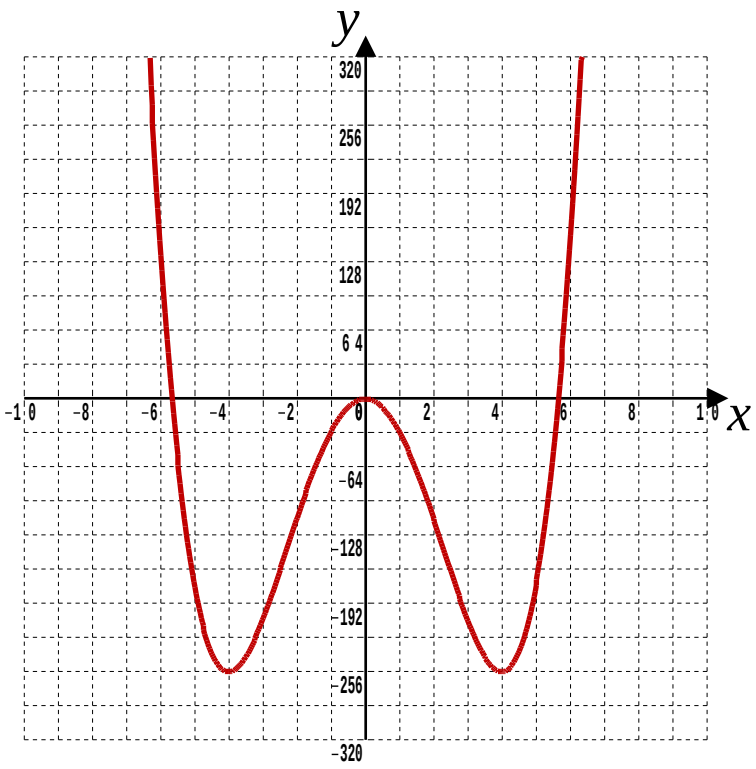
5.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 46

Question 1

The function $y = x^4 - 32x^2$ is graphed below.



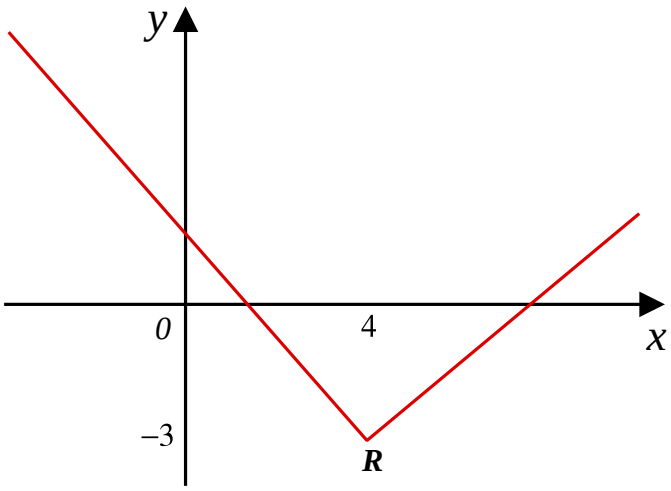
Paying attention to where the three turning points lie, sketch the related curves,

(i) $y = |x^4 - 32x^2|$ **(ii)** $y = |(x-2)^4 - 32(x-2)^2| + 32$

[2, 3 marks]

Question 2

The graph of a function, $y = f(x)$, $x \in \mathbb{R}$, consists of two line segments that meet at the point $R(4, -3)$



Sketch, on two separate diagrams, the graphs of,

(a) $y = 2f(x + 4)$

(b) $y = |f(-x)|$

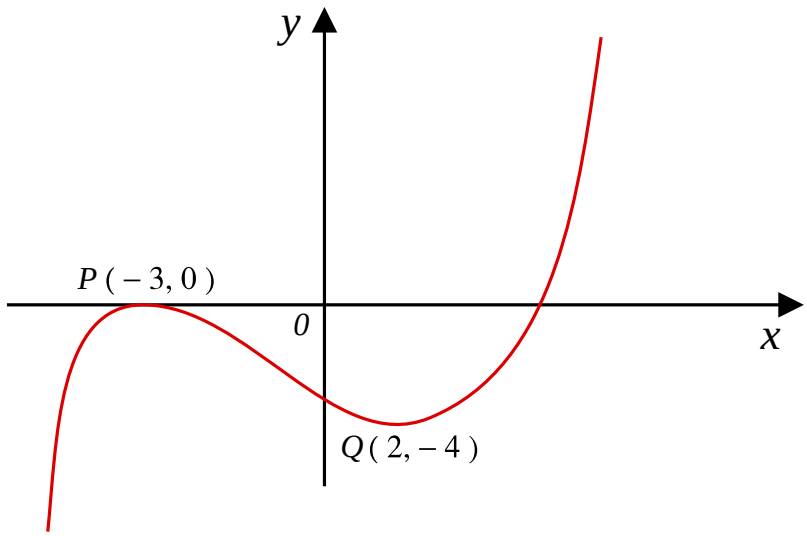
On each graph, show the coordinates of the point corresponding to R

[3, 3 marks]

Question 3

The graph of a function, $y = f(x)$, $x \in \mathbb{R}$, has two turning points.

One is at $P(-3,\ 0)$ and the other is at $Q(2, -4)$



Sketch, on two separate diagrams, the graphs of,

(a) $y = 3f(x + 2)$

(b) $y = |f(x)|$

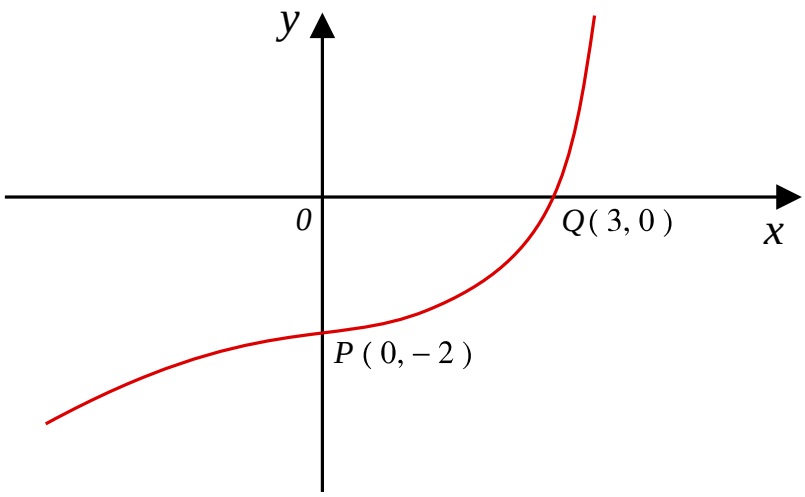
On each graph, show the coordinates of any turning points.

[3, 3 marks]

Question 4

The graph of a function, $y = f(x)$, passes through $P(0, -2)$ and $Q(3, 0)$

Furthermore, $f(x)$ is an increasing function.



Sketch, on three separate diagrams, the graphs of,

(a) $y = |f(x)|$

(b) $y = f^{-1}(x)$

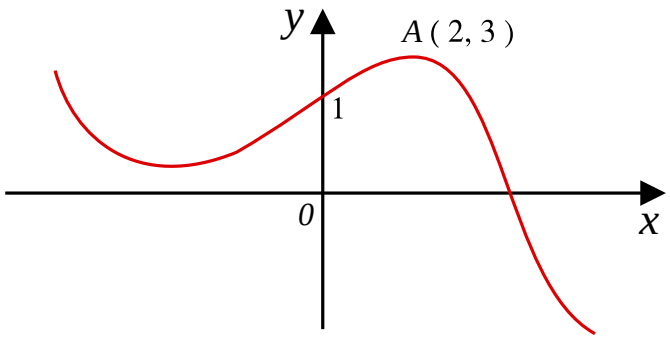
(c) $y = \frac{1}{2} f(3x)$

On each graph, show the coordinates of points where contact is made with axes.

[3, 3, 3 marks]

Question 5

The graph of a function, $y = f(x)$, $x \in \mathbb{R}$, intercepts the y-axis at $(0, 1)$ and has a local maximum at $A(2, 3)$, as shown.



Sketch, on three separate diagrams, the graphs of,

(a) $y = f(-x) + 1$

(b) $y = f(x + 2) + 3$

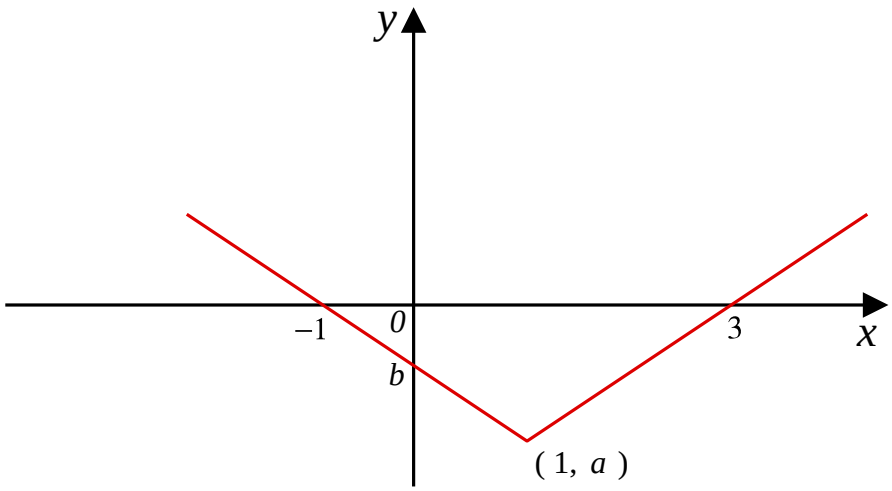
(c) $y = 2f(2x)$

On each sketch, show the coordinates of the point at which it intersects the y-axis and the coordinates of the point to which A is transformed.

[3, 3, 3 marks]

Question 6

The diagram shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$



The graph consists of two line segments that meet at the point $(1,\ a)$, $a < 0$

One line meets the x-axis at $(3,\ 0)$

The other line meets the x-axis at $(-1,\ 0)$ and the y-axis at $(0,\ b)$, $b < 0$

In separate diagrams, sketch the graph with equation,

(a) $y = f(x + 1)$

(b) $y = f(|x|)$

Indicate on each sketch the coordinates of any points of intersection with the axes.

[2, 3 marks]

Given that

$$f(x) = |x - 1| - 2$$

find

(c) the value of a and the value of b

[2 marks]

(d) the value of x for which $f(x) = 5x$

[4 marks]