Lesson 9

A-Level Pure Mathematics, Year 2

Functions II

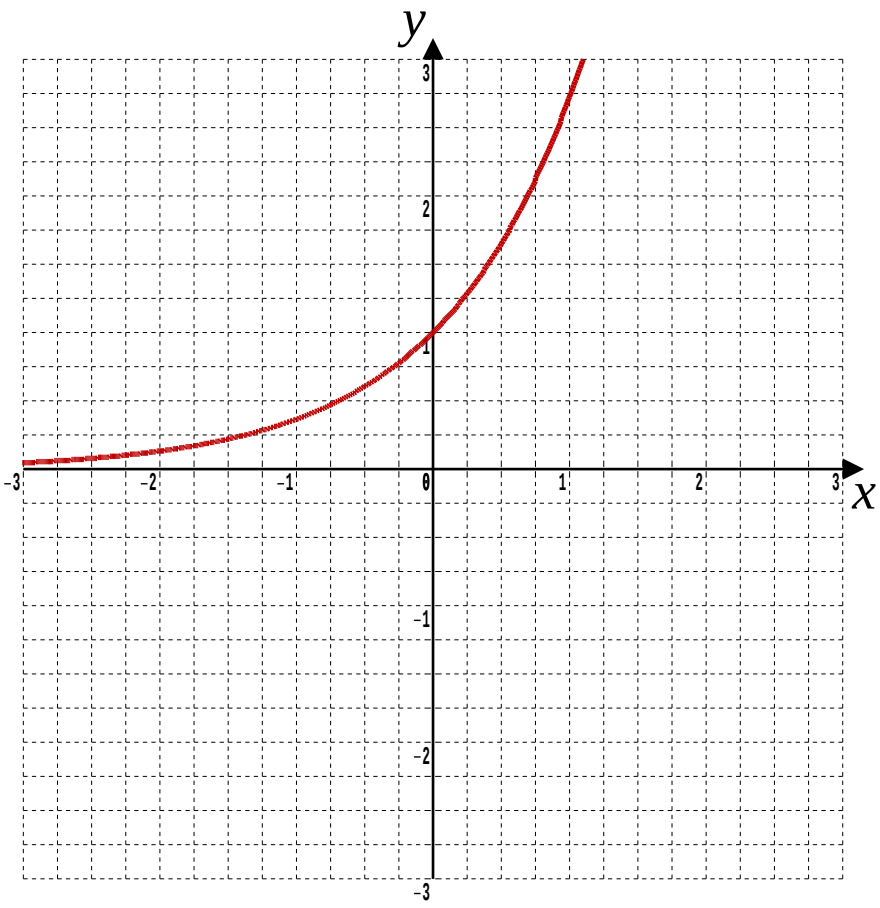
9.1 Revision

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 66

Question 1

This question is about the exponential function $f(x) = e^x, x \in \mathbb{R}$ graphed below.



(a) State the range of the function $f(x)$

[1 mark]

(b) To the graph above, add graphs of,

(i) $y = e^x + 1$ **(ii)** $y = x$ **(iii)** $y = \ln x$

[3 marks]

Question 2

Given that $f(x) = e^{3\ln x}$, $x \in \mathbb{R}$, $x > 0$, solve the equation, $f(x) = 64$

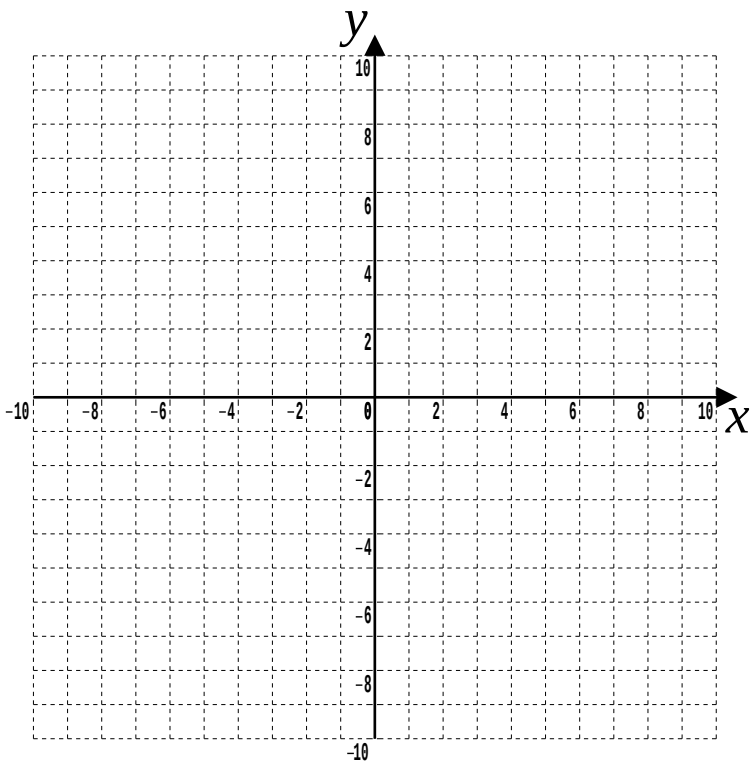
[2 marks]

Question 3

$$f(x) = |3x - 1| \qquad x \in \mathbb{R}$$

(i) Sketch the graph of $y = f(x)$ on the grid below, labelling its vertex and any points of intersection with the coordinate axes.

[3 marks]



$$g(x) = |3x - 1| - 6 \qquad x \in \mathbb{R}$$

(ii) Sketch the graph of $y = g(x)$ on the grid above, labelling its vertex and any points of intersection with the coordinate axes.

[4 marks]

(iii) Using algebra, find the coordinates of the points of intersection of

$$y = |3x - 1| - 6 \qquad \text{and} \qquad y = -\frac{1}{3}x + 3$$

[6 marks]

(iv) Add a line to the graph showing that your part (iii) answers are correct

[1 mark]

Question 4

$$f(x) = ln(x - 4), \quad x \in \mathbb{R}, \ x > 4$$

$$g(x) = e^{3x} + 4, \qquad x \in \mathbb{R}$$

(i) Find $fg(x)$, expressing the answer in simplified form, and state its range.

[3 marks]

(ii) Solve $fg(x) = 21$

[1 mark]

Question 5

$$p(x) = e^{2x} - 25, \qquad x \in \mathbb{R}$$

$$q(x) = ln(x - 3), \qquad x \in \mathbb{R}, \ x > 3$$

(i) Find $pq(x)$, expressing the answer in simplified form, and state its range.

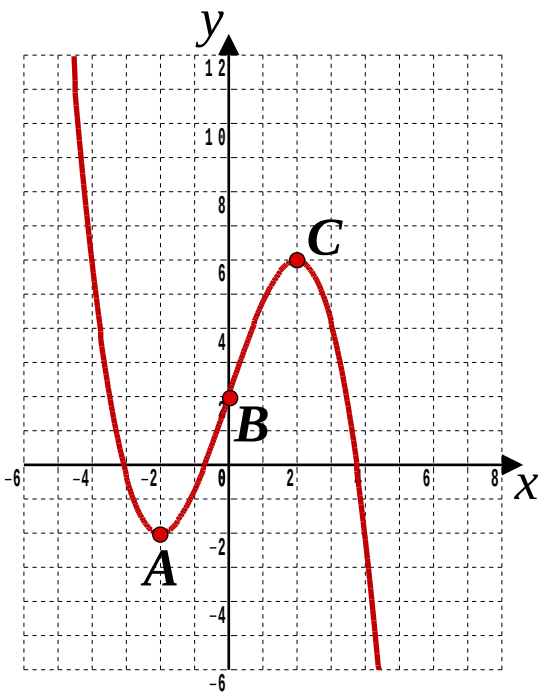
[5 marks]

(ii) Solve $pq(x) = 0$

[2 marks]

Question 6

The graph is of a mystery function $m(x)$



The points $A(-2, -2)$ and $C(2, 6)$ are turning points on the graph which also passes through the y-axis at $B(0, 2)$

Sketch on separate diagrams the graphs of,

(i) $y = |m(x)|$ **(ii)** $y = m(|x|)$ **(iii)** $y = 2m(x - 2)$

Where possible, label clearly the transformations of the points A, B and C on your diagrams and give their coordinates.

[3, 3, 4 marks]

Question 7

A-Level Examination Question from June 2018, Paper 2, Q1 (Edexcel)

$$g(x) = \frac{2x + 5}{x - 3} \qquad x \geqslant 5$$

(a) Find $gg(5)$

[2 marks]

(b) State the range of g

[1 mark]

(c) Find $g^{-1}(x)$, stating its domain.

[3 marks]

Question 8

A-Level Practice Paper from 2018, Set 2, Paper 1, Q8 (CGP)

Given that, $f^{-1}(x) = \dfrac{2x - 5}{x}, \quad x \neq 0$

$g(x) = \sqrt{2x - k}, \quad x \geqslant \dfrac{k}{2}$, where k is a positive constant

(a) find $fg(x)$, giving your answer in terms of x, and state its domain.

[3 marks]

(b) If $gg(10) = 2$, find the value of k

[3 marks]

Question 9

A-Level Examination Question from November 2017, Paper C34, Q9 (Edexcel)

$$f(x) = 2\ln(x) - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Sketch, on separate diagrams, the curve with equation,

(i) $y = f(x)$

(ii) $y = |f(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

[5 marks]

(b) Find the exact solutions of the equation $|f(x)| = 4$

[4 marks]

$$g(x) = e^{x+5} - 2, \quad x \in \mathbb{R}$$

(c) Find $gf(x)$, giving your answer in its simplest form.

[3 marks]

(d) Hence, or otherwise, state the range of gf

[1 mark]