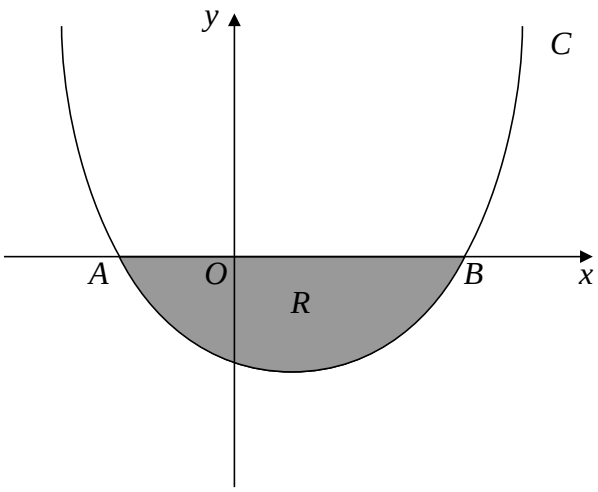
2.1 When Below the *x*-axis

In Question 10 of Exercise 1.6 we looked at the curve

$$y = (x + 1)(x - 5)$$



The following two statements are both true;

◊ $\int_{-1}^{5} (x + 1)(x - 5)\ dx = -36$

◊ $Area = +36$

If a question asks for the value of an integral, simply do the mathematics without worrying about any places where the curve is below the *x*-axis, and give the answer without any alteration to its sign.

If a question asks for an area, you must make any areas under the *x*-axis positive, before summing all positive areas to give the overall area.

2.2 A "Negative Area" Example

Teaching Video : http://www.NumberWonder.co.uk/v9043/2.mp4

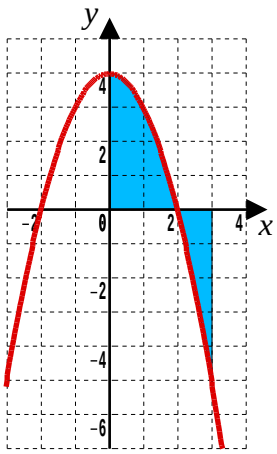
<= The video will talk through the example on the next page

Find the area bounded by $y = 4 - x^2$, the x-axis, and lines $x = 0$ and $x = 3$

$$y = 4 - x^2$$
$$= (2 + x)(2 - x)$$

So curve crosses x-axis at $x = -2$ and $x = 2$

Also, it crosses y-axis at 4



$$\int_0^2 4 - x^2 \, dx = \left[4x - \frac{x^3}{3} \right]_0^2$$
$$= \left[8 - \frac{8}{3} \right] - [0]$$
$$= \left[\frac{24 - 8}{3} \right] - [0]$$
$$= \left[\frac{16}{3} \right] - [0]$$
$$= \frac{16}{3}$$

$$\int_2^3 4 - x^2 \, dx = \left[4x - \frac{x^3}{3} \right]_2^3$$
$$= [12 - 9] - \left[\frac{16}{3} \right]$$
$$= \left[\frac{9}{3} \right] - \left[\frac{16}{3} \right]$$
$$= -\frac{7}{3}$$

$$\therefore Area = \frac{16 + 7}{3} = \frac{23}{3} = 7\frac{2}{3}$$

[6 marks]

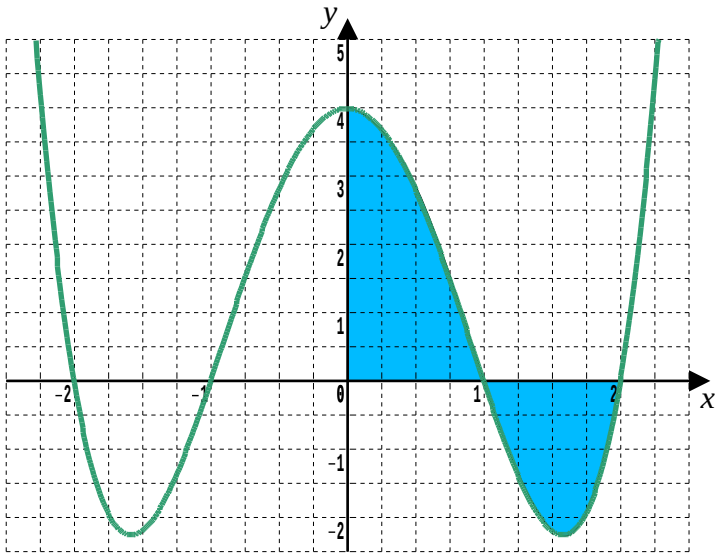
2.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 40

Question 1

The graph is of the quartic curve $y = x^4 - 5x^2 + 4$



(i) Show that $\int_1^2 y\,dx = -\frac{22}{15}$

[5 marks]

(ii) Determine the exact value of $\int_0^1 y\,dx$

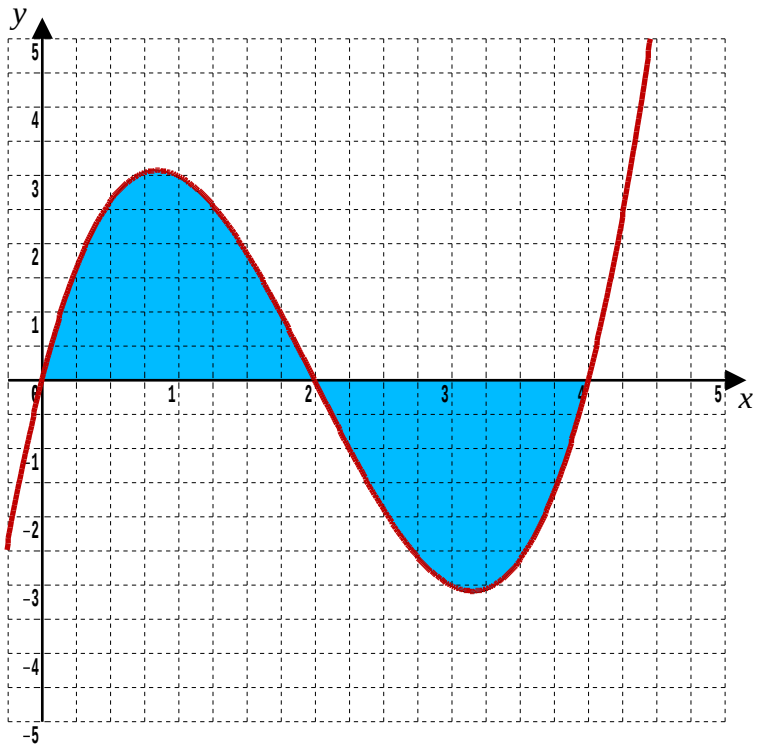
[2 marks]

(iii) Hence state the exact area that has been shaded on the graph

[1 mark]

Question 2

The graph is of the cubic curve $y = x(x - 2)(x - 4)$



(i) Without considering the graph, determine the value of $\int_0^4 y\,dx$

[4 marks]

(ii) Now consider the graph.
From your part (i) answer what can you deduce about the relationship between the area shown shaded above the x-axis and the area shown shaded below the x-axis ?

[1 mark]

Question 3

Determine the value of $\int_1^4 6x^2 - 5x^4 \, dx$

You should get a negative integer answer.

[4 marks]

Question 4

(i) Find the value of the upper limit that makes the following statement true;

$$\int_1^a (5 - 2x) \, dx = 0$$

[4 marks]

(ii) Give a geometric explanation of the part (i) result

[2 marks]

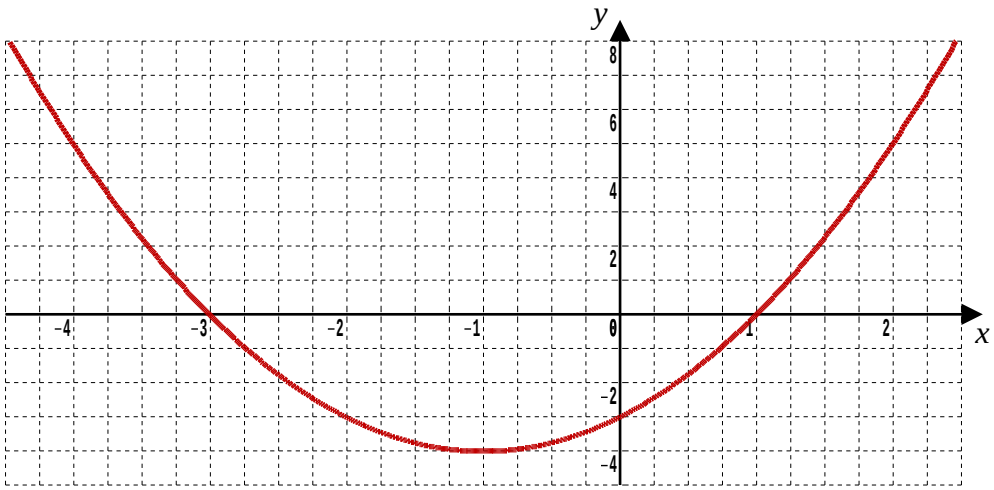
Question 5

Additional Mathematics Examination Question from June 2012, Q8 (OCR)

(i) Show that $\int_0^2 x^2 + 2x - 3 \, dx = \frac{2}{3}$

[3 marks]

The diagram shows part of the curve $y = x^2 + 2x - 3$



(ii) Marc claims that the total area between the curve, the x-axis and the lines $x = 0$ and $x = 2$ is $\frac{2}{3}$. Explain why he is wrong.

[1 mark]

(iii) Calculate total area between curve, x-axis and lines $x = 0$ and $x = 2$

[3 marks]

Question 6

This question is about using integration to find the area bounded by the curve $y = 3x - x^2$ and the x-axis and the vertical lines $x = 0$ and $x = 6$

(i) Sketch the graph of the curve and use your sketch to explain why

$$Area \neq \int_0^6 3x - x^2 \, dx$$

[4 marks]

(ii) Set up the correct integrations and evaluate them to find the area specified.

[6 marks]