Lesson 5

A-Level Pure Mathematics, Year 1
Additional Mathematics
Integration I

5.1 Without Limits

In this lesson the focus switches from “Definite” to “Indefinite” integration.

◊ ***Definite*** Integration questions are those that involving limits.
A calculator can be used to check the numerical answer.

◊ ***Indefinite*** Integrations are the algebra part only, without any limits.

With “The Fundamental Theorem of Calculus” in mind (that integration is the reverse of differentiation) an issue that immediately arises is that illustrated by the following two statements;

The derivative of $f(x) = 4x^3 + 7x + \mathbf{3}$ is $f'(x) = 12x^2 + 7$

The derivative of $g(x) = 4x^3 + 7x + \mathbf{8}$ is $g'(x) = 12x^2 + 7$

When going the other way, in other words trying to find $\int 12x^2 + 7\,dx$, how can it be decided if the answer has an isolated **+ 3** or an isolated **+ 8** ?

The answer is that, without further information, there is no way of knowing what the isolated number should be. Traditionally, the letter c is used to represent an isolated unknown number.
Thus,

$$\int 12x^2 + 7\ dx = 4x^3 + 7x + c$$

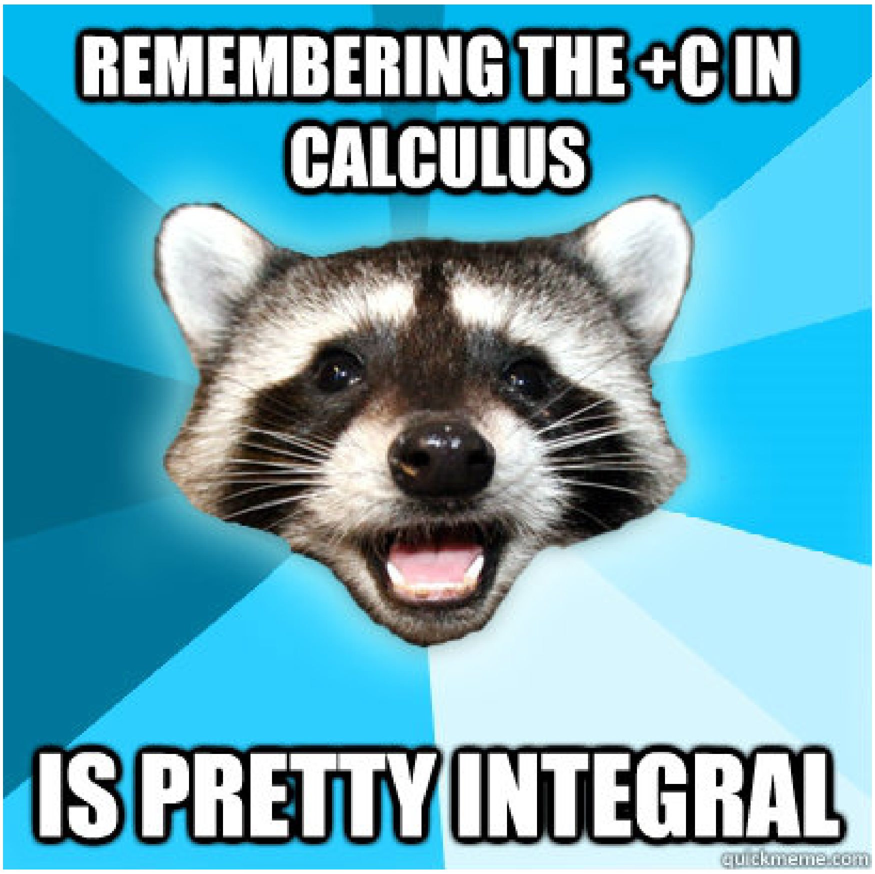
5.2 You Try

Work out the following indefinite integrations.
The answers are over the page to that you can self-check your answers.

(i) $\int 15x^4\,dx$

(ii) $14 \int x^{\frac{3}{4}}\,dx$

(iii) $\frac{3}{2} \int \frac{1}{x^4}\,dx$



5.3 You Try Answers

(i) $\int 15x^4\,dx = \dfrac{15x^5}{5} + c$

$= 3x^5 + c$

(ii) $14\int x^{\frac{3}{4}}\,dx = 14 \times \dfrac{4x^{\frac{7}{4}}}{7} + c$

$= 8x^{\frac{7}{4}} + c$

(iii) $\dfrac{3}{2}\int \dfrac{1}{x^4}\,dx = \dfrac{3}{2}\int x^{-4}\,dx$

$= \dfrac{3}{2} \times \dfrac{x^{-3}}{(-3)} + c$

$= -\dfrac{1}{2x^3} + c$

Tick which applies,

☐ MATHS = Mental Abuse To Homo Sapiens

☐ I forgot the **+ *c***

☐ I smashed it, 100%, give me more !

5.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 60

Question 1

Find the following integrals.

Simplify answers were possible.

Do not have any negative powers in your answers.

(i) $\int x^4 \; dx$

(ii) $\int 8\,x^3 \; dx$

(iii) $\int 6\,x^{-3}\, dx$

(iv) $\int \frac{5}{x^2}\, dx$

(v) $\int x^{\frac{3}{5}} \; dx$

(vi) $\int 12\,\sqrt{x} \; dx$

(vii) $\int x + 5 \; dx$

(viii) $\int \frac{x^3 + x^2}{x}\, dx$

[16 marks]

Question 2

A-Level Examination question from May 2012, Paper C1, Q1 (Edexcel)

Find

$$\int \left(6x^2 + \frac{2}{x^2} + 5 \right) dx$$

giving each term in its simplest form.

[4 marks]

Question 3

A-Level Examination Question from January 2012, Q1 (Edexcel)

Given that

$$y = x^4 + 6x^{\frac{1}{2}}$$

find in their simplest form

(a) $\dfrac{dy}{dx}$

[3 marks]

(b) $\int y \; dx$

[3 marks]

Question 4

A-Level Examination Question from May 2011, Paper C1, Q2 (Edexcel)

Given that

$$y = 2x^5 + 7 + \frac{1}{x^3} \qquad x \neq 0$$

find in their simplest form

(a) $\dfrac{dy}{dx}$

[3 marks]

(b) $\int y \, dx$

[4 marks]

Question 5

A-Level Examination Question from January 2011, Q2 (Edexcel)

Find

$$\int \left(12x^5 - 3x^2 + 4x^{\frac{1}{3}} \right) dx$$

giving each term in its simplest form.

[5 marks]

Question 6

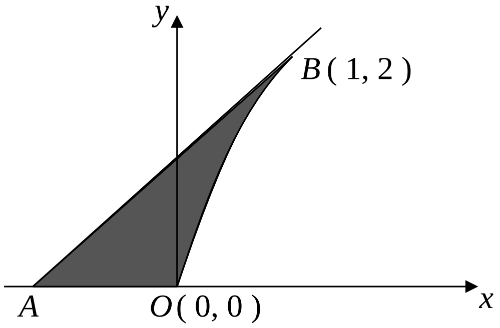
Additional Mathematics Examination Question from June 2003, Q11 (OCR)

The shaded region on the diagram shows a boat's sail.

The units are metres and, referred to the axes shown, the coordinates of O and B are (0, 0) and (1, 2) respectively.

OB is part of the curve $y = 3x - x^2$

The tangent to the curve at B meets the x-axis at A.



Find

(i) $\frac{dy}{dx}$ for the curve $y = 3x - x^2$

[2 marks]

(ii) the equation of the tangent at B

[3 marks]

(iii) the coordinates of the point A

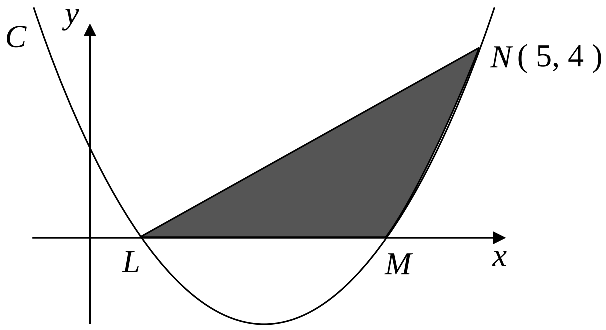
[1 mark]

(iv) the area of the sail

[6 marks]

Question 7

A-Level Examination Question from January 2010, Paper C2, Q7 (Edexcel)



The curve C has equation $y = x^2 - 5x + 4$

It cuts the x-axis at the points L and M as shown.

(a) Find the coordinates of the point L and the point M.

[2 marks]

(b) Show that the point N (5, 4) lies on C.

[1 mark]

(c) Find $\int x^2 - 5x + 4 \; dx$

[2 marks]

The region bounded by LN, LM and the curve C is shown shaded.

(d) Use your answer to part (c) to find the exact area of the shaded region.

[5 marks]