

4.1 An Identity

When a trigonometric equation is solved, such as,

$$15 \sin^2 x - 11 \sin x + 2 = 0, \quad 0^\circ \leq x \leq 360^\circ$$

the task is in finding the few values of x that make the equation true.

For this example, those values are, $x = 19.5^\circ, 23.6^\circ, 156.4^\circ, 160.5^\circ$ †

A trigonometric identity is a very different beast : It is true for all values of x .
The perfect example of a trigonometric identity is the following:

A Key Trigonometric Identity

$$\cos^2 x + \sin^2 x = 1$$

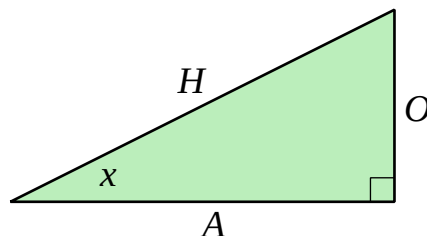
Proof

Consider a right angled triangle with sides labelled;

O for Opposite angle x

A for Adjacent to angle x

H for Hypotenuse



$$\begin{aligned}
 \text{LHS} &= \cos^2 x + \sin^2 x \\
 &= \left(\frac{A}{H}\right)^2 + \left(\frac{O}{H}\right)^2 \\
 &= \frac{A^2}{H^2} + \frac{O^2}{H^2} \\
 &= \frac{A^2 + O^2}{H^2} \quad \text{By Pythagoras' Theorem, } A^2 + O^2 = H^2 \\
 &= \frac{H^2}{H^2} \\
 &= 1 \\
 &= \text{RHS} \quad \square
 \end{aligned}$$

† This equation was solved in Lesson 2, Exercise 2.2, Question 3

4.2 Example

Solve the following trigonometric equation;

$$5 \sin x = 2 \cos^2 x - 4 \qquad 0^\circ \leq x \leq 360^\circ$$

Teaching Video : <http://www.NumberWonder.co.uk/v9044/4.mp4>



[6 marks]

4.3 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 40

Question 1

A-Level Examination Question from January 2010, Paper C2, Q2 (Edexcel)

(a) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

[2 marks]

(b) Solve, for $0^\circ \leq x \leq 360^\circ$

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

[4 marks]

Question 2

Find all the solutions, in the interval $0 \leq x \leq 360^\circ$, of the equation

$$\cos^2 x - 3 \cos x = \sin^2 x + 4$$

[6 marks]

Question 3

The next most useful trigonometric identity after $\cos^2 x + \sin^2 x = 1$ is;

Another Key Trigonometric Identity

$$\frac{\sin x}{\cos x} = \tan x \quad (\text{ Provided } \cos x \neq 0)$$

Your task now is to prove that this result is true.

Your proof will be in a similar style to that for $\cos^2 x + \sin^2 x = 1$

To get you started here are the first few parts of the proof;

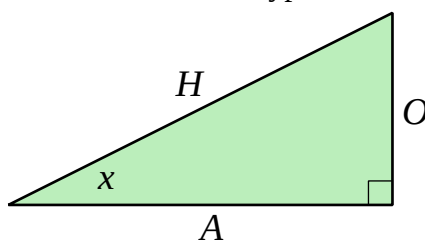
Proof

Consider a right angled triangle with sides labelled;

O for Opposite angle x

A for Adjacent to angle x

H for Hypotenuse



$$\text{LHS} = \frac{\sin x}{\cos x}$$

Complete the proof, started above.

[3 marks]

Question 4

Additional Mathematics Examination Question from June 2014, Q9 (OCR)

(i) Show that $\frac{1 - \cos^2 x}{1 - \sin^2 x} = \tan^2 x$

[1 mark]

(ii) Hence solve the equation $\frac{1 - \cos^2 x}{1 - \sin^2 x} = 3 - 2 \tan x$ for values of x in the range $0^\circ \leq x \leq 180^\circ$

[4 marks]

Question 5

A-Level Examination Question from October 2016, Paper C12, Q10 (edited) (Edexcel)

(a) Given that

$$8 \tan x = -3 \cos x$$

show that

$$3 \sin^2 x - 8 \sin x - 3 = 0$$

[3 marks]

(b) Hence solve, for $0 \leq x < 360^\circ$,

$$8 \tan x = -3 \cos x$$

giving your answers to one decimal place, as appropriate.

[5 marks]

Question 6

A-Level Examination Question from January 2005, Paper C2, Q4 (Edexcel)

(a) Show that the equation

$$5 \cos^2 x = 3(1 + \sin x)$$

can be written as

$$5 \sin^2 x + 3 \sin x - 2 = 0$$

[2 marks]

(b) Hence solve, for $0^\circ \leq x \leq 360^\circ$, the equation

$$5 \cos^2 x = 3(1 + \sin x)$$

giving your answers to 1 decimal place where appropriate.

[5 marks]

Question 7

A-Level Examination Question from January 2011, Paper C2, Q7 (Edexcel)

(a) Show that the equation

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

can be written in the form

$$4 \sin^2 x + 7 \sin x + 3 = 0$$

[2 marks]

(b) Hence solve, for $0^\circ \leq x \leq 360^\circ$

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

[5 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com