

2.1 Thinking Backwards

When $y = \frac{(4 + 5x)^6}{6}$ is differentiated the result is

$$\begin{aligned}\frac{dy}{dx} &= \frac{6(4 + 5x)^5}{6} \times 5 && \text{by the chain rule} \\ &= 5(4 + 5x)^5\end{aligned}$$

When $y = \frac{[f(x)]^{n+1}}{(n+1)}$ is differentiated the result is

$$\begin{aligned}\frac{dy}{dx} &= \frac{(n+1)[f(x)]^n}{(n+1)} \times f'(x) && \text{by the chain rule} \\ &= f'(x) [f(x)]^n\end{aligned}$$

By The Fundamental Theorem of Calculus,

The Chain Rule Backwards

$$\int f'(x) [f(x)]^n dx = \frac{[f(x)]^{n+1}}{(n+1)} + c \quad n \neq -1$$

The two key consequences of this result are;

- Every time a new integration question is tackled, a mental scan must be made to see if it is in the form of a function raised to a power with the derivative of that function sitting in front.

In other words, watch out for $f'(x) [f(x)]^n$

- An alertness needs to be maintained for situations in which the desired set up of $f'(x) [f(x)]^n$ can be created by making use of a “fiddle factor”. See example N° 2 and example N° 3.

Example N° 1

Determine: $\int 7(7x + 2)^3 dx$

[3 marks]

Solution : It is spotted that with $f(x) = 7x + 2 \Rightarrow f'(x) = 7$ and that this situation is that of a chain rule backwards with $n = 3$

$$\text{Thus, } \int 7(7x + 2)^3 dx = \frac{(7x + 2)^4}{4} + c$$

The teaching video will talk through Example N° 2 and Example N° 3

Teaching Video: <http://www.NumberWonder.co.uk/v9045/2.mp4>



Example N° 2

Determine: $\int (3x + 1)^4 dx$

[3 marks]

Example N° 3

Determine: $\int \frac{24}{(4x - 2)^4} dx$

[3 marks]

2.2 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 50

In each question use The Chain Rule Backwards to perform the integration given.
Most questions will require that a “fiddle factor” be introduced.

Question 1

$$\int (3x + 2)^5 dx$$

[3 marks]

Question 2

$$\int (7x - 3)^3 dx$$

[3 marks]

Question 3

$$60 \int (4 + 5x)^3 dx$$

[3 marks]

Question 4

$$\int (3 - 2x)^5 dx$$

[3 marks]

Question 5

$$\int \left(7 - \frac{1}{2} x \right)^6 dx$$

[3 marks]

Question 6

$$18 \int \sqrt{1 + 4x} dx$$

[3 marks]

Question 7

$$\int \left(\frac{2x}{3} + 8 \right)^{\frac{1}{2}} dx$$

[3 marks]

Question 8

$$\int \frac{1}{(2x - 1)^4} dx$$

[3 marks]

Question 9

$$\int 6 (2x - 1)^{-2} dx$$

[3 marks]

Question 10

$$\int \frac{6}{\sqrt{3x - 1}} dx$$

[3 marks]

Question 11

$$\int \frac{1}{4 (x + 3)^2} dx$$

[3 marks]

Question 12

$$\int 14 (3 + 2x)^{-2} dx$$

[3 marks]

Question 13

$$\int \frac{1}{(1 - 2x)^{\frac{3}{2}}} dx$$

[3 marks]

Question 14

$$\int_0^1 (4x + 1)^4 dx$$

[3 marks]

Question 15

$$\int_0^4 \sqrt{2x + 1} dx$$

[4 marks]

Question 16

$$\int_2^5 \frac{1}{(x - 1)^3} dx$$

[4 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com