

3.1 Unlocking The Backwards

The Chain Rule Backwards

$$\int f'(x) [f(x)]^n dx = \frac{[f(x)]^{n+1}}{(n+1)} + c \quad n \neq -1$$

Initial problems involving The Chain Rule Backwards kept $f(x)$ as a simple linear function. That is, $f(x) = mx + c$
However, this limitation will now be removed !

Example N° 1

Determine: $\int 120x^3(3x^4 + 7)^4 dx$

Teaching Video: <http://www.NumberWonder.co.uk/v9045/3a.mp4>



Example N° 1 is all that's needed for the first six questions in Exercise 3.2
So you may prefer to do those questions first then watch the Example N° 2
video ahead of Question 7 onwards.

Example N° 2

Determine: $\int 64(x^3 - 2)(x^4 - 8x)^3 dx$

Teaching Video: <http://www.NumberWonder.co.uk/v9045/3b.mp4>



[4 marks]

3.2 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 40

In each question use The Chain Rule Backwards to perform the integration given.
Most questions will require that a “fiddle factor” be introduced.

Question 1

Determine: $\int 150x(3x^2 + 1)^4 dx$

[3 marks]

Question 2

Determine: $\int 56x(x^2 - 5)^3 dx$

[3 marks]

Question 3

Determine: $\int 42x^2(2 - x^3)^6 dx$

[3 marks]

Question 4

Determine: $\int 12x\sqrt{x^2 + 6} dx$

[3 marks]

Question 5

Determine: $\int \frac{21x}{(1+x^2)^2} dx$

[3 marks]

Question 6

Determine: $\int \frac{12x^3}{\sqrt{3+x^4}} dx$

[3 marks]

Question 7

Determine: $\int 60(x^2 - 1)(x^3 - 3x + 7)^4 dx$

[4 marks]

Question 8

Determine: $\int \frac{x + 1}{(x^2 + 2x + 3)^4} dx$

[4 marks]

Question 9

Determine: $\int \frac{x^2 + 2}{\sqrt{x^3 + 6x}} dx$

[4 marks]

Question 10

(i) Differentiate $y = \sin(x^3 + 9x)$

[3 marks]

(ii) With part (i) in mind, find $\int (x^2 + 3) \cos(x^3 + 9x) dx$

[3 marks]

(iii) Determine: $\int x^2 \cos(4x^3 - 7) dx$

[4 marks]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School

It may be freely duplicated and distributed, unaltered, for non-profit educational use

In October 2020, Shrewsbury School was voted “**Independent School of the Year 2020**”

© 2025 Number Wonder

Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com