

5.1 Partial Fractions : The Comeback

The “special case” of The Chain Rule Backwards, when $n = -1$, occurs more often than might initially be expected. This is because complicated algebraic fraction integrations are handled by first rewriting them using partial fractions with the resulting integrations often being the $n = -1$ case.

Here is an example to give an overall idea of how partial fractions, used at the start, gives rise to “special case” integrations. Don't worry about the details as these will be revised shortly. This first example is just to give a feel for the direction of travel.

Example N° 1

$$\begin{aligned}\int \frac{4x + 1}{(x + 1)(x - 2)} dx &= \int \frac{1}{x + 1} dx + \int \frac{3}{x - 2} dx \\ &= \int (x + 1)^{-1} dx + 3 \int (x - 2)^{-1} dx \\ &= \ln|x + 1| + 3 \ln|x - 2| + c\end{aligned}$$

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“To show you how well I understand fractions,
I only did half of my homework.”

~ Only a fraction of you will find this funny ~

The Chain Rule Backwards

$$\int f'(x) [f(x)]^n dx = \frac{[f(x)]^{n+1}}{(n+1)} + c \quad n \neq -1$$

$$\int f'(x) [f(x)]^{-1} dx = \ln|f(x)| + c \quad \text{i.e. with } n = -1$$

Example N° 2

Determine $\int \frac{9x^2}{(6x+1)(3x+1)^2} dx$

Teaching Video: <http://www.NumberWonder.co.uk/v9045/5a.mp4> (Part 1)
<http://www.NumberWonder.co.uk/v9045/5b.mp4> (Part 2)



<= Part 1

Part 2 =>



[8 marks]

5.2 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 40

Question 1

- (i) Find the value of A and the value of B , given that,

$$\frac{10x + 19}{(2x + 1)(x + 4)} = \frac{A}{(2x + 1)} + \frac{B}{(x + 4)}$$

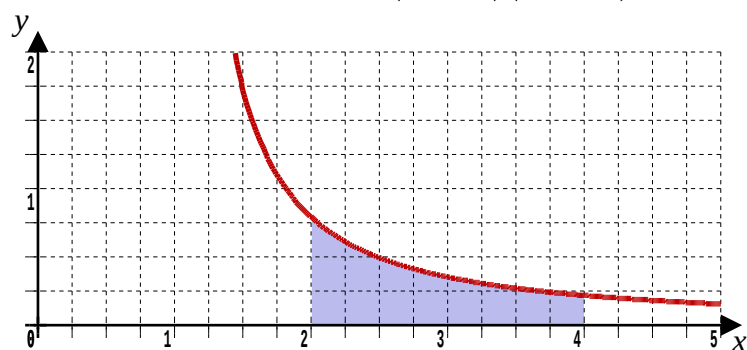
[4 marks]

- (ii) Hence show that $\int_0^1 \frac{10x + 19}{(2x + 1)(x + 4)} dx = \ln\left(\frac{3^2 \times 5^3}{2^6}\right)$

[6 marks]

Question 2

The graph is on the function, $f(x) = \frac{(x + 2)}{(x - 1)(2x + 1)}$



Show that $\int_2^4 f(x) dx = \frac{1}{2} \ln 5$

[10 marks]

Question 3

(i) Find the values of A , B and C given that,

$$\frac{1}{(2x - 1)x^2} = \frac{A}{(2x - 1)} + \frac{B}{x} + \frac{C}{x^2}$$

[4 marks]

(ii) Hence show that $\int_1^2 \frac{1}{(2x - 1)x^2} dx = \ln\left(\frac{9}{4}\right) - \frac{1}{2}$

[6 marks]

Question 4

A-Level Examination Question from June 2012, Paper C4, Q1

$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A , B and C

[4 marks]

(b) (i) Hence find

$$\int f(x) \, dx$$

[3 marks]

- (ii) Find $\int_1^2 f(x) dx$ leaving your answer in the form $a + \ln b$
where a and b are constants

[3 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com