

6.1 Beyond The Power Function

Suitable integrals, I , that can be determined by applying The Chain Rule Backwards have been spotted by identifying the key situation;

$$I = a \int f'(x) [f(x)]^n dx \quad \text{where } n \text{ and } a \text{ are constants}$$

That power of n can be thought of as a function in its own right.

With $g(x) = x^n$ another way of expressing what we've been looking for is;

$$I = a \int f'(x) g[f(x)]$$

There is no reason why function g has to be a power function; The Chain Rule Backwards can be wielded when g is other functions such as $\cos$ or $\sin$

The Chain Rule Backwards

$$\int f'(x) [f(x)]^n dx = \frac{[f(x)]^{n+1}}{(n+1)} + c \quad n \neq -1$$

$$\int f'(x) [f(x)]^{-1} dx = \ln|f(x)| + c \quad \text{i.e. with } n = -1$$

$$\int f'(x) \cos[f(x)] dx = \sin[f(x)] + c$$

$$\int f'(x) \sin[f(x)] dx = -\cos[f(x)] + c$$

$$\int f'(x) e^{[f(x)]} dx = e^{[f(x)]} + c$$

Example

Determine: $\int 63x^2 e^{7x^3} dx$

Teaching Video: <http://www.NumberWonder.co.uk/v9045/6.mp4>



6.2 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 40

Question 1

Determine $\int 30x e^{5x^2} dx$

[3 marks]

Question 2

Evaluate: $\int_0^2 x^2 e^{x^3} dx$

Give the answer to the nearest integer.

[4 marks]

Question 3

Determine: $\int 100 x^3 \cos(6 + 5 x^4) dx$

[4 marks]

Question 4

Determine: $\int \frac{3}{2} x \sin(x^2) dx$

[4 marks]

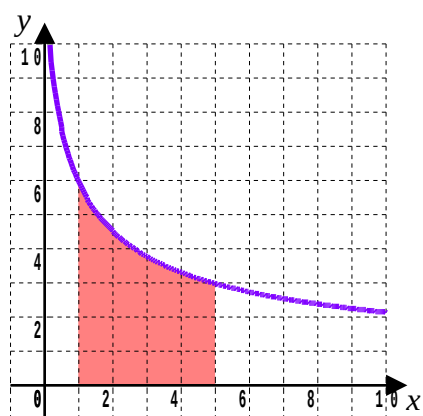
Question 5

Determine : $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$

[4 marks]

Question 6

The graph is of the function $f(x) = \frac{12}{\sqrt{3x+1}}$

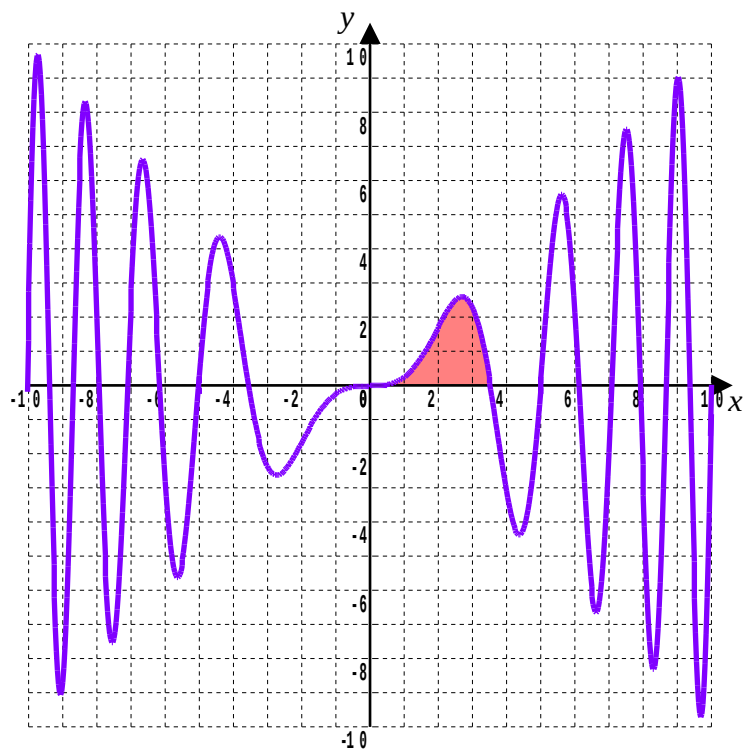


Find the area shown shaded in the graph which is bound by the curve, the x-axis and the vertical lines $x = 1$ and $x = 5$

[7 marks]

Question 7

The graph is of the function, $g(x) = x \sin\left(\frac{x^2}{4}\right)$



Find the exact area of the 'bump' shown shaded in the graph which is bounded by $g(x)$, the x -axis and the lines $x = 0$ and $x = 2\sqrt{\pi}$

[7 marks]

Question 8

$$f(x) = e^{\sqrt{x}}$$

- (i) Find $f'(x)$

[2 marks]

- (ii) Evaluate to 3 decimal places;

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

[5 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com