Year 1

~ MECHANICS ~

Constant Acceleration Kinematics

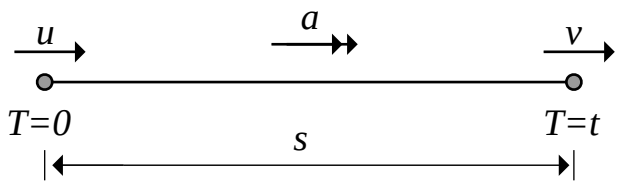
K • I • N • E • M • A • T • I • C • S

Lesson 1

Constant Acceleration Kinematics
Applied Mathematics (Mechanics) Year 1

1.1 Derivation of the SUVAT equations

Consider a straight line interval over which a particle accelerates uniformly;



s = *displacement*

u = *initial velocity*

v = *final velocity*

a = *acceleration* (*constant*)

t = *time*

We know that acceleration is is the rate of change of velocity.

Furthermore, if the acceleration is constant,

$$a = \frac{change\ in\ velocity}{change\ in\ time}$$

$$a = \frac{v - u}{t - 0}$$

$$at = v - u$$

$$\therefore\ v = u + at \qquad \textit{Result } N^\circ 1$$

When the acceleration is zero, we know that

$$Distance = Speed \times Time$$

but when there is constant acceleration this becomes

$$Displacement = Average\ Speed \times Time$$

$$s = \left(\frac{v + u}{2}\right) t \qquad \textit{Result } N^\circ 2$$

Substituting *Result N°* 1 into *Result N°* 2 gives *Result N°* 3 as follows;

$$s = \left(\frac{v + u}{2} \right) t$$

$$s = \left(\frac{(u + at) + u}{2} \right) t$$

$$s = \left(\frac{2u + at}{2} \right) t$$

$$s = \left(\frac{2u}{2} + \frac{at}{2} \right) t$$

$$s = \left(u + \frac{1}{2} at \right) t$$

$$s = ut + \frac{1}{2} a t^2 \qquad \textit{Result N°} 3$$

A fourth result is obtained by first noting that *Result N°* 1

$$v = u + at$$

can be written as

$$t = \frac{v - u}{a}$$

and substituting this into *Result N°* 2

$$s = \left(\frac{v + u}{2} \right) t$$

$$s = \left(\frac{v + u}{2} \right) \left(\frac{v - u}{a} \right)$$

$$s = \frac{v^2 - u^2}{2a}$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as \qquad \textit{Result N°} 4$$

A fifth result is obtained by first noting that *Result N°* 1

$$v = u + at$$

can be written as

$$u = v - at$$

and substituting this into *Result N°* 2

$$s = \left(\frac{v + u}{2} \right) t$$

$$s = \left(\frac{v + v - at}{2} \right) t$$

$$s = \left(\frac{2v - at}{2} \right) t$$

$$s = \left(\frac{2v}{2} - \frac{at}{2} \right) t$$

$$s = \left(v - \frac{1}{2} at \right) t$$

$$s = vt - \frac{1}{2} a t^2 \qquad \textit{Result N°} 5$$

1.2 The five SUVAT equations

$v = u + at$	s
$s = vt - \frac{1}{2} a t^2$	u
$s = ut + \frac{1}{2} a t^2$	v
$s = \left(\frac{v + u}{2} \right) t$	a
$v^2 = u^2 + 2as$	t

1.3 Example

Additional Mathematics Examination Question from June 2010, Q6 (corrected)

An aeroplane touches down at a point *A* on a runway, travelling with a velocity of 90 m s^{-1}. It then decelerates uniformly until it reaches a velocity of 6 m s^{-1} at a point *B* on the runway, 2016 m from *A*.

(i) Find the deceleration.

[3 marks]

(ii) Find the time taken to travel from *A* to *B*.

[2 marks]

1.4 Exercise

Question 1

Additional Mathematics Examination Question from June 2007, Q5 (corrected)

A car is travelling along a motorway with a velocity of 30 m s^{-1}. At the moment that it passes a point *A* the brakes are applied so that the car decelerates with constant deceleration.

When it reaches a point *B*, where *AB* = 300 m, the velocity of the car is 10 m s^{-1}.

Calculate

(i) the constant deceleration

[3 marks]

(ii) the time taken to travel from *A* to *B*.

[2 marks]

Question 2

Additional Mathematics Examination Question from June 2009, Q5

Parcels slide down a ramp.

Due to resistance the deceleration is 0.25 m s^{-2}

(i) One parcel is given an initial velocity of 2 m s^{-1}
Find the distance travelled before the parcel comes to rest.

[3 marks]

(ii) A second parcel is given an initial velocity of 3 m s^{-1} and takes 4 seconds to reach the bottom of the ramp.
Find the length of the ramp.

Be careful : The parcel will be moving as it flies off the end of the ramp !

[3 marks]

Question 3

Without looking them up, try to write out all five *suvat* equations from memory.

Question 4

Additional Mathematics Examination Question from June 2003, Q10 (corrected)

A car, initially travelling with velocity 20 m s^{-1}, accelerates uniformly at 1.2 m s^{-2}.

Find

(i) the speed after 5 seconds

[2 marks]

(ii) the distance travelled in this time.

[2 marks]

Question 5

A particle moving in a straight line experiences a constant retardation of 6 $m.s^{-2}$
It passes a point, *A*, with initial velocity 25 $m.s^{-1}$
As it passes *A* a stopwatch is started.

(i) What will the stopwatch read as the particle first has a displacement of 28 metres from *A* ?

(ii) What will the stopwatch read when the particle has a displacement of 28 metres from *A* for the second time ?

(iii) There is a third moment when the particle is at a ***distance*** of of 28 metres from *A.*
Determine the stopwatch reading for this third moment.