Lesson 5

Constant Acceleration Kinematics
Applied Mathematics (Mechanics) Year 1

5.1 Elaborate *suvat* Problems

An Investigation

A *sky spy* police camera is launched vertically upwards at 49m/s.
Use

$$s = ut + \frac{1}{2}at^2$$

to complete the left hand table showing its height at 0.5 second intervals for the first seven seconds of its flight.

$s = ?$ *find this*

$u = 49$

$v = \ldots$ *not relevant*

$a = -9.8$

$t = 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7.$

Time, t_1	Height
0	0
0.5	23.3
1	44.1
1.5	62.5
2	78.4
2.5	91.9
3	102.9
3.5	111.5
4	
4.5	
5	
5.5	
6	
6.5	
7	

Time, t_2	Height
0	0
0.5	23.3
1	44.1
1.5	62.5
2	78.4
2.5	91.9
3	102.9
3.5	111.5
4	
4.5	
5	
5.5	

A second *sky spy* camera is accidentally launched (with the same initial velocity) from the same launcher (and so the same flight path) 1.5 seconds after the first. Complete the right hand table to show its height at 0.5 second intervals.

Identify the interval in which the two *sky spy* cameras collide.

A Mathematical Approach

In the investigation into the 'alleged incompetence' that led to the collision, it was required that the precise moment and height at which the two *sky spy* cameras collided be calculated.
(A building was damaged and a small child hurt when they fell back to Earth unable to open their parachutes).

The basic method with this sort of problem is to calculate the time first, and then find the displacement.

The first step is this;

$$u\,t_1 + \frac{1}{2}a\,t_1^{\,2} = u\,t_2 + \frac{1}{2}\,a\,t_2^{\,2}$$

But the **key** next step is this;

$$u\,t_1 + \frac{1}{2}\,a\,t_1^2 = u\,(t_1 - 1.5) + \frac{1}{2}\,a\,(t_1 - 1.5)^2$$

Or simply;

$$u\,t + \frac{1}{2}\,a\,t^2 = u(t - 1.5) + \frac{1}{2}\,a\,(t - 1.5)^2$$

if we keep in mind that *t* is the time into the first *sky spy* launch.

Solve the equation to calculate the moment of impact, and then the corresponding height.

5.2 Exercise

Question 1

Suppose the first *sky spy* camera had been launched at 98m/s and the second at 49m/s and 16 seconds after the first.

Calculate **(i)** The time of collision.

(ii) The height at which collision occurs.

Question 2

Some bank robbers are driving at 17 ms^{-1}, with constant acceleration 1 ms^{-2}, as they pass a stationary police car parked in a layby.

The police start to chase them 8 seconds later, accelerating uniformly at 2 ms^{-2}.

When and how far from the layby do the police catch up with the robbers ?

Question 3

A train passes through station A at 45ms^{-1}, with constant acceleration 0.5 ms^{-2}, heading east towards station B which is 1375 metres away.

Five seconds later, another train passes through station B at a constant speed of 25 ms^{-1}, travelling west.

Where do the trains pass each other ?

Question 4

A car travelling along a straight level road at a constant speed of 54 km/h passes a second car as it starts to accelerate from rest at a uniform rate of $0.5ms^{-2}$.

Find the time that elapses and the distance covered when the second car draws level with the first.

Question 5

A ball is thrown vertically upwards from a point A with a speed of 20 ms^{-1}.
At the same instant a second ball is dropped from a point B which is 60 m vertically above A. Find the time which elapses before the two balls meet and their height above A at this instant.

Question 6

A stone is dropped from the top of a tower.

After 1 second, another stone is thrown vertically downwards from the same point at a speed of 15 ms^{-1}.

If the stones reach the ground simultaneously, find the height of the tower.