Lesson 7

Constant Acceleration Kinematics
Applied Mathematics (Mechanics) Year 1

7.1 Velocity - Time Graphs #2

On a Velocity - Time graph

◊ The gradient represents acceleration.

Furthermore

$$a = \frac{\Delta v}{\Delta t} \qquad \left(acceleration = \frac{change\ in\ velocity}{change\ in\ time} \right)$$

◊ Area under the graph represents distance

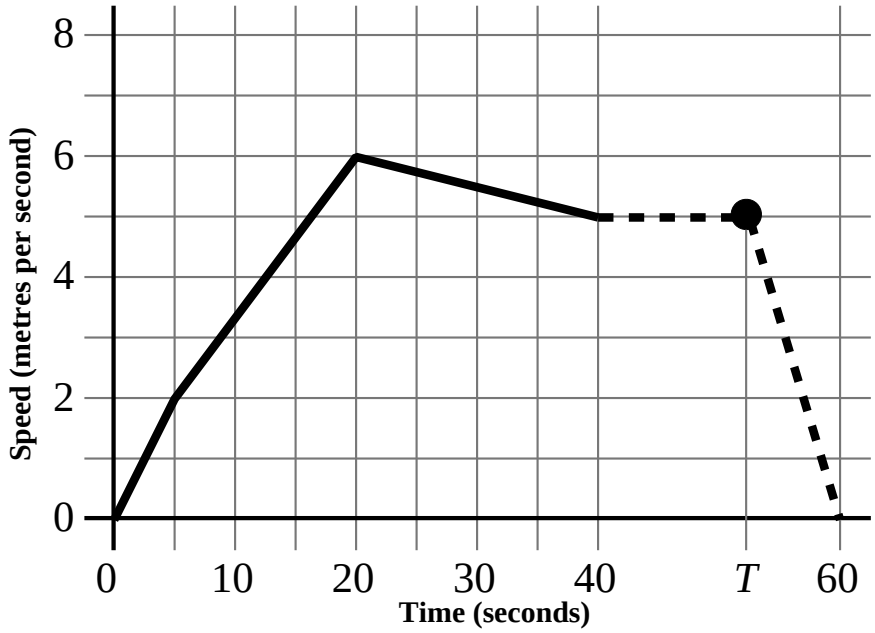
An Example

The velocity - time graph is of an orienteer's progress as she runs between two controls on a straight route.

The distance between the controls is 240 metres.

The velocity-time graph shows her

- starting from rest
- (constant) acceleration for 5 seconds to a speed of 2 ms^{-1}
- (constant) acceleration for 15 seconds to a speed of 6 ms^{-1}
- (constant) deceleration for 20 seconds to a speed of 5 ms^{-1}
- moving with constant velocity for (T - 40) seconds
- (constant) deceleration for (60 - T) seconds
- stopping
- taking 60 seconds in total



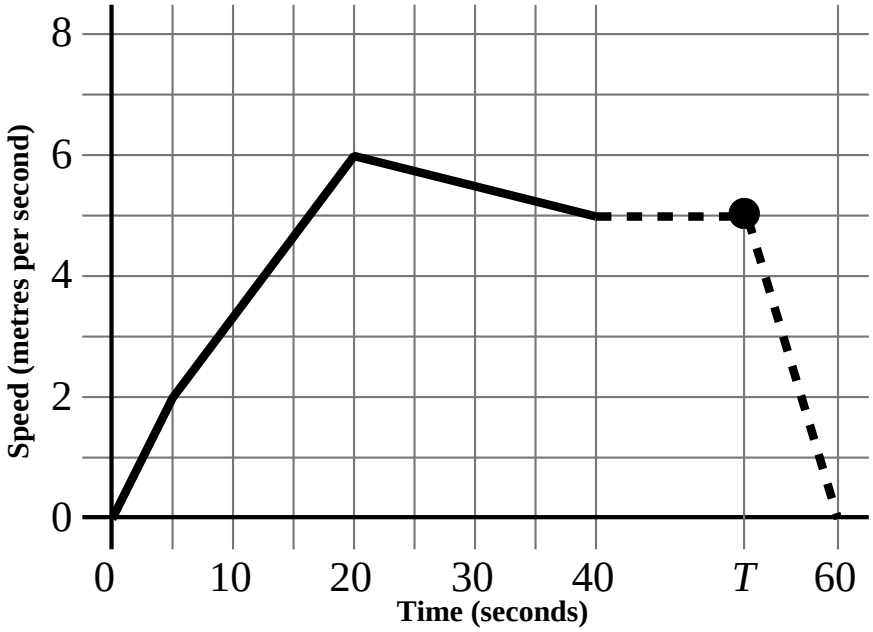
(i) What is the acceleration between $t = 5$ and $t = 20$ seconds ?

(ii) Determine the value of T

Solution to The Example

(i)

(ii)



7.2 Exercise

Question 1

M1 Examination Question, November 2002, Q3

A car accelerates uniformly from rest to a speed of 20 ms^{-1} in T seconds. The car then travels at a constant speed of 20 ms^{-1} for $4T$ seconds and finally decelerates uniformly to rest in a further 50 s.

(a) Sketch a speed-time graph to show the motion of the car.

[2 marks]

The total distance travelled by the car is 1220 m

(b) Find the value of T

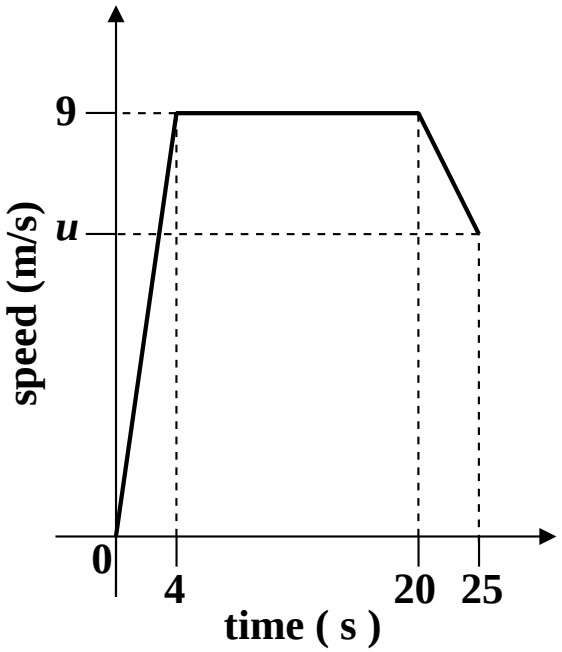
[3 marks]

(c) Find the initial acceleration of the car.

[2 marks]

Question 2

M1 Examination Question, January 2005, Q3



A sprinter runs a race of 200 m.
Her total time for running the race is 25 s.
The sketch is of the speed-time graph for the motion of the sprinter.
She starts from rest and accelerates uniformly to a speed of 9 ms^{-1} in 4 s.
The speed of 9 ms^{-1} is maintained for 16 s and she then decelerates uniformly to a speed of u ms^{-1} at the end of the race.

Calculate

(a) the distance covered by the sprinter in the first 20 s of the race,

[2 marks]

(b) the value of u,

[4 marks]

(c) the deceleration of the sprinter in the last 5 s of the race

[3 marks]

Question 3

M1 Examination Question, June 2005, Q5

A train is travelling at 10 ms^{-1} on a straight horizontal track.
The driver sees a red signal 135 m ahead and immediately applied the brakes.
The train immediately decelerates with constant deceleration for 12 s, reducing its speed to 3 ms^{-1}.
The driver then releases the brakes and allows the train to travel at a constant speed of 3 ms^{-1} for a further 15 s.
He then applies the brakes again and the train slows down with constant deceleration, coming to rest as it reaches the signal.

(a) Sketch a speed-time graph to show the motion of the train

[3 marks]

(b) Find the distance travelled by the train from the moment when the brakes are first applied to the moment when its speed first reaches 3 ms^{-1}

[2 marks]

(c) Find the total time from the moment when the brakes are first applied to the moment when the train comes to rest.

[5 marks]

Question 4

M1 Examination Question, January 2012, Q6

A car moves along a straight horizontal road from a point A to a point B.

$AB = 885$ m

The car accelerates from rest at A to a speed of 15 ms^{-1} at a constant rate a ms^{-2}

The time for which the car accelerates is $\frac{1}{3}T$ seconds.

The car maintains the speed of 15 ms^{-1} for T seconds.

The car then decelerates at a constant rate of 2.5 ms^{-2} stopping at B.

(a) Find the time for which the car decelerates

[2 marks]

(b) Sketch a speed-time graph for the motion of the car

[2 marks]

(c) Find the value of T

[4 marks]

(d) Find the value of a

[2 marks]

(e) Sketch an acceleration-time graph for the motion of the car

[3 marks]

Question 5

M1 Examination Question, January 2004, Q6

A train starts from rest at a station *A* and moves along a straight horizontal track.
For the first 10 s the train moves with constant acceleration 1.2 ms^{-2}
For the next 24 s it moves at a constant acceleration 0.75 ms^{-2}
It then moves with constant speed for *T* seconds.
Finally it slows down with constant deceleration 3 ms^{-2} until it comes to a rest at station *B*

(a) Show that, 34 s after leaving *A*, the speed of the train is 30 ms^{-1}

[3 marks]

(b) Sketch a speed-time graph to illustrate the motion of the train as it moves from *A* to *B*

[3 marks]

(c) Find the distance moved by the train during the first 34 s of its journey from A

[4 marks]

The distance from A to B is 3 km

(d) Find the value of T

[4 marks]