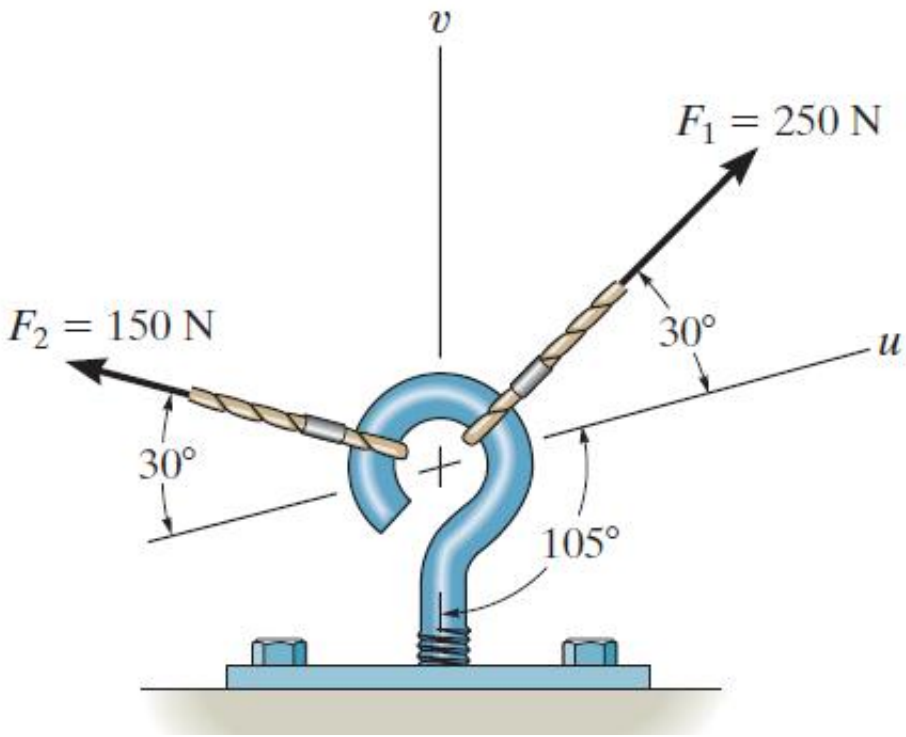
A-Level Applied Mathematics
Mechanics Year 2

S T A T I C S



S T A T I C S

Lesson 1

A-Level Applied Mathematics
Mechanics : Statics : Year 2

1.1 Analysis Of A Static Particle

Statics is about particles that are not moving; neither changing location nor rotating. Given a particle that is static, by Newton's first law of Motion, in any given direction the resultant force acting on a particle must be zero.

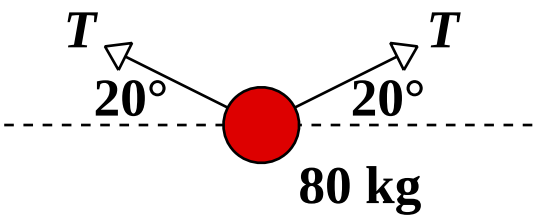
A typical analysis of a two dimensional statics problem begins by considering the particle to be at the origin of a horizontal *x*-axis and vertical *y*-axis. Any force on the particle that is not along an axis is resolved into two component parts, one component being along the *x*-axis and the other along the *y*-axis.

The analysis continues by balancing forces along the *x*-axis and, separately, balancing forces along the *y*-axis.

From an algebraic interplay between the two resulting equations, a typical statics problem can be solved.

1.2 Man On A Wire

As an example of a statics problem consider the situation of an 80 kg man on a wire. Assume the wire has no weight and does not stretch; the wire is light and inextensible. The wire makes an angle of 20° to the horizontal on either side of the man.



By symmetry the tension in the wire, *T*, has the same magnitude, either side.

The task now is to determine the magnitude of *T*

(i) Resolve one of two forces, *T*, into a horizontal and a vertical component part.

(ii) Assume the man is a particle.
Draw a diagram showing all significant forces acting on the particle.

(iii) By balancing forces vertically, determine the magnitude of *T*

An Observation
The man has a weight of $80 \times 9.8 = 784$ Newtons

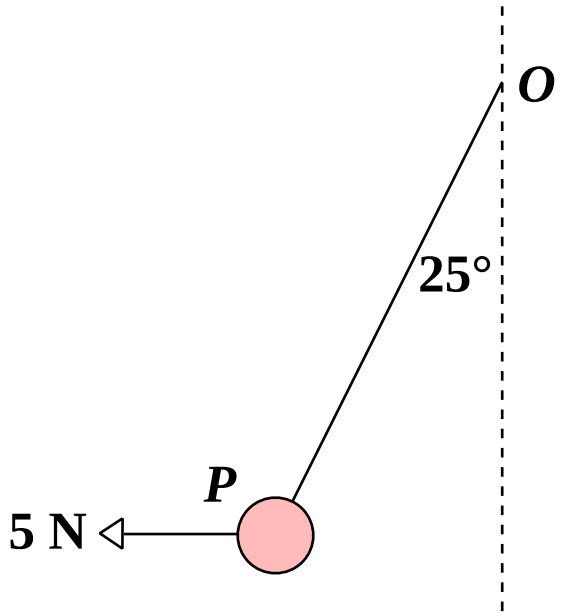
Is the tension in the rope more, the same, or less than this ?

What does this imply about the strength of the wire needed by a tight rope walker ?

1.3 Exercise

Question 1

A-Level Examination Question from June 2014, M1(R), Q1



A particle of weight W newtons is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point, O.
A horizontal force of magnitude 5 N is applied to P.
The particle P is in equilibrium with the string taut and with OP making an angle of 25° to the downward vertical, as shown.

Find,

(a) the tension in the string

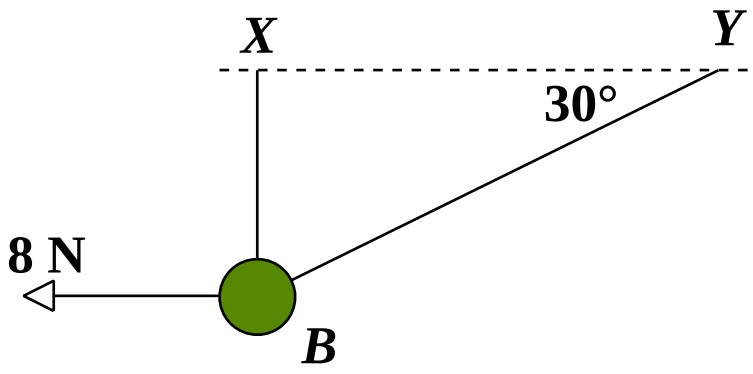
[3 marks]

(b) the value of W

[3 marks]

Question 2

A smooth bead, *B*, of mass *m* is threaded on a light inextensible string.
The ends of the string attached to two fixed points, *X* and *Y*, on a horizontal ceiling.
The bead is held in equilibrium by a horizontal force of magnitude 8 Newtons which is acting parallel to *YX*.
The motionless bead, *B*, is vertically below *X* and $\angle BYX$ is 30°, as shown.



FYI : As the bead is smooth the tension, *T*, in the string *BX* is the same as in string *BY*

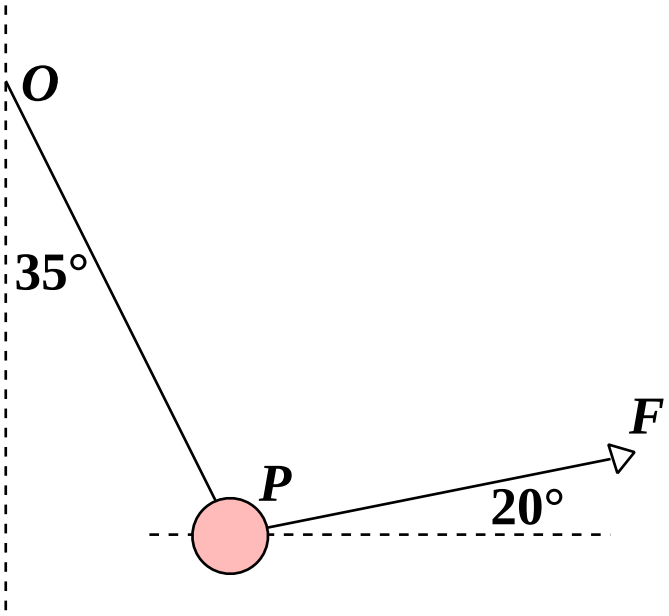
(i) Draw a diagram of the particle and the forces acting upon it. Any force that is not horizontal or vertical should be resolved into component parts that are.

(ii) Find the tension is the string

(iii) What is the mass of the bead, *m* ?

Question 3

A particle *P* of mass 3 kg is attached to one end of a light string, the other end of which is attached to a fixed point *O*. The particle is held in equilibrium, with *OP* at 35° to the downward vertical, by a force of magnitude *F* newtons. The force acts in the same vertical plane as the string and acts at an angle of 20° to the horizontal.



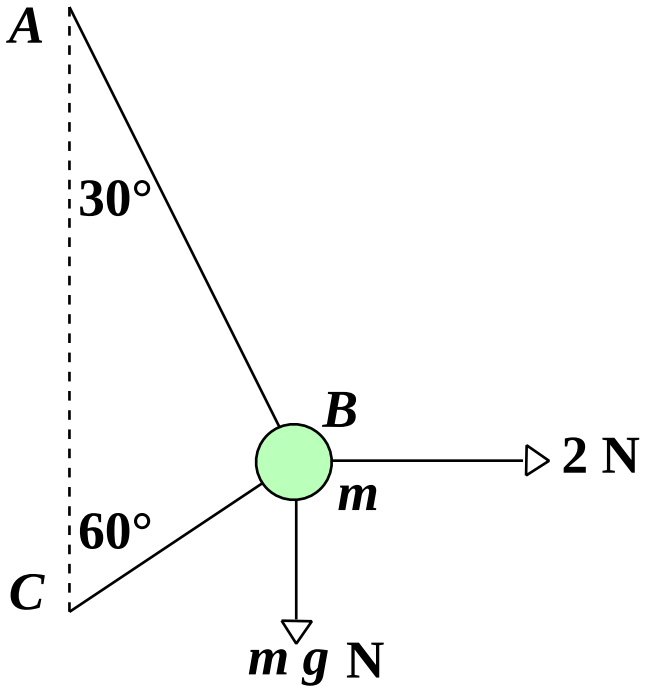
(i) The force *F* can be resolved into two component parts.
Is *F cos* 20 the horizontal or the vertical component ?

(ii) The tension in the string, *T*, is also resolved into two component parts.
Is *T cos* 35 the horizontal or the vertical component ?

(iii) By balancing forces horizontally and vertically obtain two separate equations and then combine these to determine the value of *F* and the value of *T*

Question 4

A smooth bead *B* is threaded on a light inextensible string. The ends of the string are attached to two fixed points *A* and *C* where *A* is vertically above *C*. The bead is held in equilibrium by a horizontal force of magnitude 2 N. The sections *AB* and *BC* of the string make angles of 30° and 60° with the vertical respectively.



Find

(a) the tension in the string

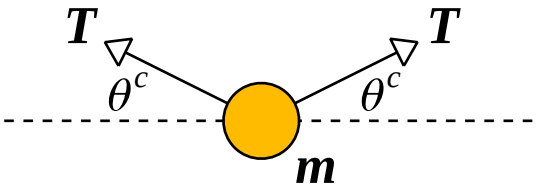
[3 marks]

(b) the mass of the bead, giving your answer to the nearest gram

[4 marks]

Question 5

A tight rope walking woman of mass m kg is stationary at the centre of a wire rope which makes an angle of θ radians to the horizontal on either side of her.



(i) Prove that the tension, T, in the wire rope is given by,

$$T = \frac{1}{2} mg \csc \theta$$

(ii) By working out $\frac{dT}{d\theta}$ in terms of m, g and θ find the value of θ for which the tension in the rope is a minimum.